Mechanism design in network markets[1]

Journée COSMOS

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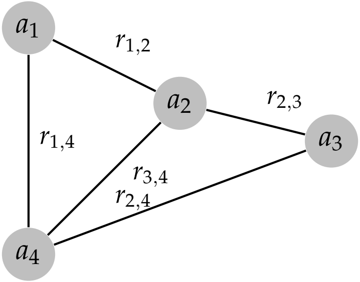
[1] Joint work with Alejandro Jofré and Nicolas Figueroa

Electricity markets

(Part B of my thesis)

My three objectives for the presentation

- Present the market model.
- Convince you that in this model producers exercise market power.
- Show you how we can deal with this market power using mechanism design. The final formulation we get is surprisingly simple.



- Figueroa, Jofré, B. H. : Cost minimizing regulation for a wholesale electricity market, (to be submitted).
- B. H., Jofré: Mechanism design and auctions for electricity network (2016)
- B. H., Jofré: Mechanism design and allocation algorithms for network markets with piece-wise linear costs and quadratic externalities, (to be submitted).

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The study of electricity markets brings together different fields of research

- **Markets modeling:** Anderson, Jofré, Escobar, Henrion, Philpott, Aussel, Hu, Ralph ...
- **Mechanism design** Aussel, Laffont, Guesnerie, Martimort, Tirole, Milgrom, Myerson, Groves ...
- **Bayesian game theory** Athey, Berger, Edlin, Shannon, Topkis, McAdams, Vives, Reny ...
- **Key ideas can be found in:** [Escobar and Jofré, 2010], [Myerson, 1981], [Topkis, 1998]

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The actors of the market

: The ISO minimizes supply cost s.t. supply meet demand

: a producer wants to make some profit

: another producer competes with (b)

Figure: Actors of the market

- Each producer i bids a marginal price b_i
- The ISO decides the production allocation $q_i(b)$ so that supply $\geq$ demand and total cost is minimized
- Producers get paid x according to their bid and the production they had to supply individually (pay as bid market): $x_i(b) = b_i q_i(b)$
- Electricity can travel through the lines, but quadratic loss

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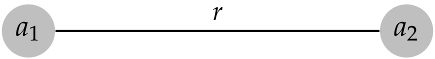
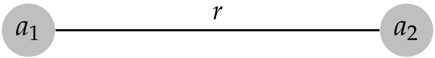
The binodal example



- Demand = d at each node.
- Assume the bids are b_1 and b_2.
- The operator has to decide the production levels q_1 and q_2.
- The operator can send h_{12} of electricity from 1 to 2 (loss rh_{12}^2).
- The operator can send h_{21} of electricity from 2 to 1 (loss rh_{21}^2).
- At optimality, $h_{12}h_{21} = 0$.
- At optimality electricity will flow from the cheapest node to the most expensive.

The binodal example



The ISO solves

$$\underset{q,h}{\text{minimize}}\; b_1 q_1 + b_2 q_2$$

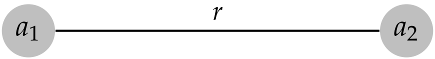
subject to:

$$q_1 - h_{12} + h_{21} \geq \frac{r}{2}[h_{12}^2 + h_{21}^2] + d$$

$$q_2 - h_{21} + h_{12} \geq \frac{r}{2}[h_{12}^2 + h_{21}^2] + d$$

$$q_i, h_i \geq 0 \text{ for } i = 1, 2$$

The binodal example



The ISO solves

$$\underset{q,h}{\text{minimize}} \; b_1 \, q_1 + b_2 \, q_2$$

subject to:

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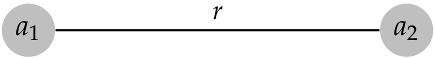

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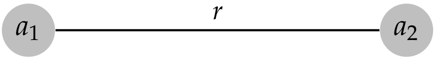

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$$q_i, h_i \geq 0 \text{ for } i = 1, 2$$

More generally, for a larger network with piecewise linear production cost functions

The ISO solves

$$\begin{aligned}
\underset{(q,h)}{\text{minimize}} \quad & \sum_{i \in \{Agents\}} \sum_{j \in \{Slopes\}} q_i^j b_i^j \\
\text{subject to} \quad & \sum_{j \in \{Slopes\}} q_i^j + \sum_{i' \in V(i)} h_{i',i} - h_{i,i'} - \frac{h_{i,i'}^2 + h_{i',i}^2}{2} r_{i,i'} \geq d_i \\
& h_{i,i'} \geq 0 \\
& q_i^j \geq 0 \\
& q_i^j \leq \bar{q}.
\end{aligned}$$

♣

More generally, for a larger network with piecewise linear production cost functions

The ISO solves

$$\underset{(q,h)}{\text{minimize}} \quad \sum_{i\in\{Agents\}} \sum_{j\in\{Slopes\}} q_i^j b_i^j$$

$$\text{subject to} \quad \sum_{j\in\{Slopes\}} q_i^j + \sum_{i'\in V(i)} h_{i',i} - h_{i,i'} - \frac{h_{i,i'}^2 + h_{i',i}^2}{2} r_{i,i'} \geq d_i$$

$$h_{i,i'} \geq 0$$

$$q_i^j \geq 0$$

$$q_i^j \leq \bar{q}.$$

♣

Solution of ISO(b) for the linear binodal setting

If q_1 and q_2 are stricly positive, then $q_1 = F(b_1, b_2)$ and $q_2 = F(b_2, b_1)$, where

$$F(\lambda_1, \lambda_2) = d + \frac{1}{2r}\left(\frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}\right)^2 - \frac{1}{r}\left(\frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}\right).$$

Solution of ISO(b) for the linear binodal setting

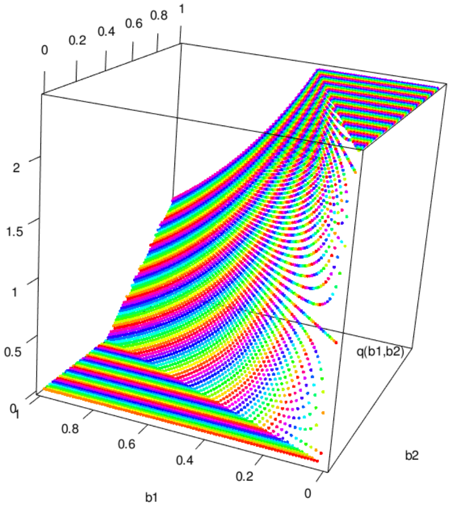
$$F(\lambda_1, \lambda_2) = d + \frac{1}{2r}\left(\frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}\right)^2 - \frac{1}{r}\left(\frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}\right) \quad \tilde{q} = 2\left[\frac{1 - \sqrt{1 - 2dr}}{r}\right]$$

Then

$$q_i(b_i, b_{-i}) = \begin{cases} F(b_i, b_{-i}) & \text{if } F(b_i, b_{-i}) \geq 0 \text{ and } F(b_{-i}, b_i) \geq 0 \\ \tilde{q} & \text{if } F(b_{-i}, b_i) < 0 \text{ and } F(b_i, b_{-i}) \geq 0 \\ 0 & \text{if } F(b_i, b_{-i}) < 0 \text{ and } F(b_{-i}, b_i) \geq 0 \end{cases}$$

And the payments are $x_i(b) = b_i q_i(b_i, b_{-i})$ (pay-as-bid).

The solution with respect to b_1 and b_2



General network

No closed form expression, but:

- In general, we show the production level at each node is a (closed form) **function of the Lagrangian multipliers** associated with the nodal constraints of the neighboring nodes.
- Those multipliers can be computed by iteration of a **fixed point monotone operator**.

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- Assume each producer has a linear production cost of slope c
- Assume that this is common knowledge
- Profit for a producer 1: $(b_1 - c)F(b_1, b_2)$
- Profit for a producer 2: $(b_2 - c)F(b_2, b_1)$
- We get the Nash equilibrium strategies:

Market power

$$b^* = \frac{c}{1 - 2dr} > c$$

What if $r = 0$?

What if $r = 0$? $\rightarrow$ bids= c, earns 0.

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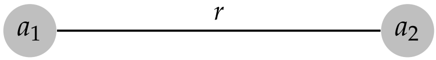
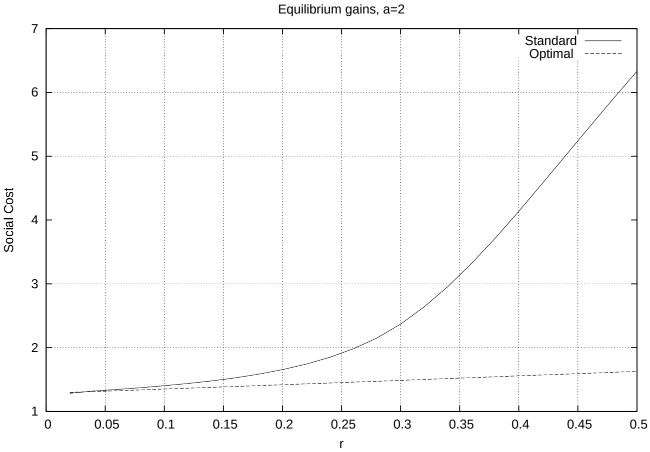
Just to recap, why are we doing this?



To minimize tax money overspending !!!

Social cost: Optimal Mechanism (doted line) vs Standard Setting



The new mechanism

- Producers bid
- (before) They receive production allocation and payments (from the ISO allocation problem) ($x_i(b) = b_i q_i(b)$)
- (in the next slides) They receive production allocation and payments (BUT a new rule $(q_i(b), x_i(b))$)

What if the ISO knew the c_i ?

The Bayesian setting

At each node i:

- There is an electricity producer whose production cost is piecewise-linear of slopes $(c_i^1, \dots, c_i^N)$ such that for a production level between $j\bar{q}$ and $(j+1)\bar{q}$, the marginal cost is c_i^j
- c_i^j is unknown to the ISO and the other producers, but they have a prior (probability distribution) $f_i^j(c_i^j)$ on it (independent).
- This probability distribution corresponds to some common knowledge on i.
- From a technical perspective, f_i^j needs to satisfy some properties to make the forthcoming construction possible.

Mechanism design

Revelation Principle

Theorem (Revelation Principle)

To any Bayesian Nash equilibrium of a game of incomplete information, there exists a payoff-equivalent direct revelation mechanism that has an equilibrium where the players truthfully report their types.

Mechanism design

Revelation Principle

Theorem (Revelation Principle)

"We can limit our search of new (x, q) *to those such that the mechanism is truthful"*

- Idea: change the ISO behaviors
- Tool: the revelation principle
- So: we perform an optimization over the truthful direct mechanisms

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Definitions

- A *direct mechanism* is a triple $(q, x, h) \in (\mathcal{Q}, \mathcal{X}, \mathbb{H})$
- BAYESIAN NASH EQUILIBRIUM= strategy profile such that each player maximizes his expected payoff given his belief about the other players' types and given the strategies played by the other players.
- $K_i^j(c_i^j) = \int_{c_i^{j-}}^{c_i^j} f_i^j(s)ds / f_i^j(c_i^j)$
- $Q_i^j(c_i) = \mathbb{E}_{-i} \min((q_i(c_i, c_{-i}) - (j-1)\bar{q})^+, \bar{q})$
- $X_i(c_i) = \mathbb{E}_{-i} x_i(c_i, c_{-i})$
- $U_i(c_i, c_i') = X_i(c_i') - \sum_{j \in J} c_i^j Q_i^j(c_i')$

The optimization problem

Problem

$$\underset{(q,x,h)}{\text{minimize}} \sum_{i \in \{Producers\}} \text{Average Payment for producer } i$$

subject to

Supply is greater than demand.

It's optimal for all producers to tell the truth.

Every producer wants to participate in the market.

The optimization problem

Problem

$$\underset{(q,x,h)}{\textit{minimize}} \sum_{i \in \{Producers\}} \mathbb{E} x_i(c)$$

subject to

supply greater than demand

It's optimal for all producers to tell the truth

Every producer wants to participate in the market.

The optimization problem

Problem

$$\underset{(q,x,h)}{minimize} \sum_{i \in \{Producers\}} \mathbb{E} x_i(c)$$

subject to

$$q_i(c) + \sum_{i' \in V(i)} h_{i',i}(c) - h_{i,i'}(c) - \frac{h^2_{i,i'}(c) + h^2_{i',i}(c)}{2} r_{i,i'} \geq d_i$$

It's optimal for all producers to tell the truth

Every producer wants to participate in the market.

The optimization problem

Problem

$$\underset{(q,x,h)}{\textit{minimize}} \sum_{i \in \{Producers\}} \mathbb{E} x_i(c)$$

subject to for all (c, c')

$$q_i(c) + \sum_{i' \in V(i)} h_{i',i}(c) - h_{i,i'}(c) - \frac{h^2_{i,i'}(c) + h^2_{i',i}(c)}{2} r_{i,i'} \geq d_i$$

$$U_i(c_i, c_i) \geq U_i(c_i, c'_i)$$

Every producer wants to participate in the market.

The optimization problem

Problem

$$\underset{(q,x,h)}{\textit{minimize}} \sum_{i \in \{Producers\}} \mathbb{E} x_i(c)$$

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$$U_i(c_i, c_i) \geq U_i(c_i, c'_i)$$

$$U_i(c_i, c_i) \geq 0.$$

The optimization problem

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$$U_i(c_i, c_i) \geq U_i(c_i, c_i') \quad \textit{HARD}$$

$$U_i(c_i, c_i) \geq 0. \quad \textit{HARD}$$

Reformulation

Theorem

The solution of the optimal mechanism satisfies (under some hypothesis displayed in the next slide)

$$\underset{(q,h)}{\textit{minimize}} \quad \mathbb{E} \sum_{i \in \{Producers\}} \sum_{j \in \{Slopes\}} q_i^j(c) \hat{c}_i^j$$

subject to for all c

$$q_i(c) + \sum_{i' \in V(i)} h_{i',i}(c) - h_{i,i'}(c) - \frac{h_{i,i'}^2(c) + h_{i',i}^2(c)}{2} r_{i,i'} \geq d_i$$

And set $x_i(c) = \sum_{j \in \{Slopes\}} q_i^j(c) c_i^j + \int_{c_i^j}^{\bar{c}_i^j} q_i^j(t; c_{-i}) dt$.

Where $\hat{c}_i^j = (c_i^j + K_i^j(c_i^j))$ *can be computed from the data.*

Remember ?

The ISO allocation problem

ISO(b)

$$\underset{(q,h)}{\text{minimize}} \quad \sum_{i \in \{Agents\}} \sum_{j \in \{Slopes\}} q_i^j b_i^j$$

$$\text{subject to} \quad \sum_{j \in \{Slopes\}} q_i^j + \sum_{i' \in V(i)} h_{i',i} - h_{i,i'} - \frac{h_{i,i'}^2 + h_{i',i}^2}{2} r_{i,i'} \geq d_i$$

The reformulation

Theorem

The solution of the optimal mechanism satisfies (under some hypothesis displayed in the next slides)

$$\underset{(q,h)}{\text{minimize}} \quad \mathbb{E} \sum_{i \in \{Producers\}} \sum_{j \in \{Slopes\}} q_i^j(c) \hat{c}_i^j$$

subject to for all c

$$q_i(c) + \sum_{i' \in V(i)} h_{i',i}(c) - h_{i,i'}(c) - \frac{h_{i,i'}^2(c) + h_{i',i}^2(c)}{2} r_{i,i'} \geq d_i$$

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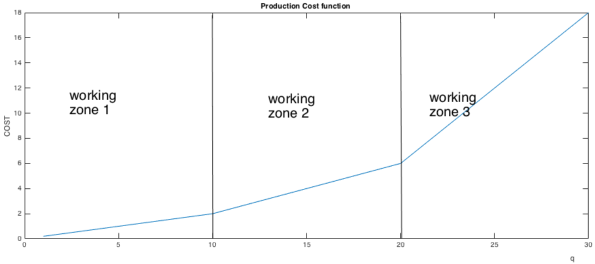
Where $\hat{c}_i^j = (c_i^j + K_i^j(c_i^j))$ *can be computed from the data.*

The assumptions on f_i



- NON OVERLAPPING WORKING ZONES: $\boldsymbol{C}_i = [c_i^{1-}, c_i^{1+}] \times \ldots \times [c_i^{N-}, c_i^{N+}]$
- DISCERNABILITY ASSUMPTION: the virtual cost $c_i^j + K_i^j(c_i^j)$ is increasing in j
- MONOTONE LIKELIHOOD RATIO PROPERTY: $c_i^j \rightarrow c_i^j + K_j^i(c_i^j)$ is increasing in c_i^j

Regularity of the allocation

Theorem

For any agent i and production segment j, $Q_i^j(c_i)$ is C^∞.

Proof.

1. Isolate the corner solutions (in the dual and primal)
2. Show that the weight of those corner solutions is smoothed out by integration

□

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On a class of Bayesian games that converges to their unique equilibrium

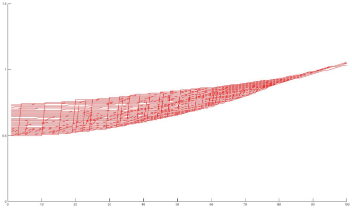

Figure: Best Reply

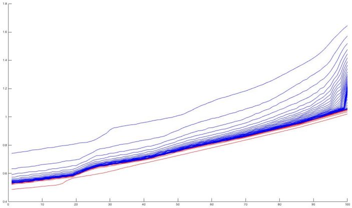

Figure: Differential Equation

Conclusion

- We solved the **optimal design problem** for a market model with piecewise-linear cost functions and general externalities.
- We derived some properties of the **solutions of the ISO allocation problem**.
- For some markets, we derived **the existence and uniqueness of the induced Bayesian Nash equilibrium**. The algorithm we introduced to compute this equilibrium showed good numerical properties wrt the best replies approach.

Next

- Extend the class of markets for the Bayesian games analysis
- Numerical simulation and theoretical benchmark for the gap

Some references

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Thank you for your attention