StochDynamicProgramming.jl : a Julia package for multistage stochastic optimization.

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Contents

Programming Language for Numerical Mathematics

We often tends to believe that making numerical code should be done in a two step process:

1. writing the algorithm in a high-level script like language (matlab, scilab, python...) to test the idea and variations on small to medium sized problem.
2. re-writing the algorithm in a low level language (C, C++, Fortran...) for numerical efficiency applicable on large scale problem.

Which means writing the algorithm twice.

Could we do it only once ?

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- A beautiful woman name
- A "high-level, high-performance language for technical computing, with syntax that is familiar to users of other technical computing environments"
- Summary of best features:
  - Supports Windows, OSX, and Linux
  - Just in Time compiler
  - Automatic generation of efficient, specialized code for different argument types
  - Designed for parallelism and distributed computation
  - Powerful shell-like capabilities for managing other processes
  - Call python and C function
  - Built-in Package manager
  - MIT licensed: free and open source
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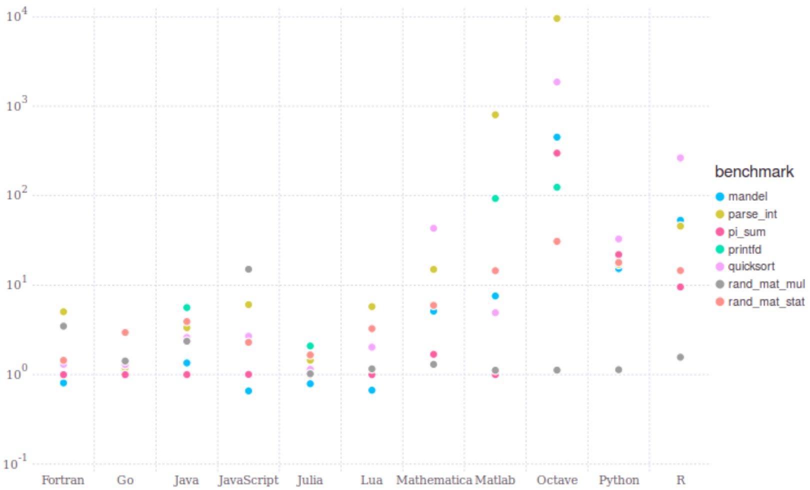
Is Julia really faster than alternatives ?



What is JuliaOpt ?

- JuliaOpt is an organization that brings together packages written in Julia that are related to optimization.
- Two modeller:
  - JuMP
  - Convex
- Interfaced with a number of solver (CPLEX, Gurobi, Mosek, Clp, GLPK, Knitro, Cbc, ECOS, Ipopt, NLopt, SCS).
- Optim contains the basic non-linear algorithms.

Contents

Problem statement

We consider the optimization problem

$$\begin{aligned} \min_{\pi} \quad & \mathbb{E}\left(\sum_{t=0}^{T-1} L_t(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_t) + K(\mathbf{X}_T)\right) \\ s.t. \quad & \mathbf{X}_{t+1} = f_t(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_t) \\ & \mathbf{U}_t = \pi_t(\mathbf{X}_t, \mathbf{W}_t) \end{aligned}$$

under the crucial assumption that $(\mathbf{W}_t)_{t\in\{1,\cdots,T\}}$ is a white noise

Stochastic Dynamic Programming

By the white noise assumption, this problem can be solved by dynamic programming, where the Bellman functions satisfy

$$\begin{cases} V_T(x) & = & K(x) \\ \hat{V}_t(x, w) & = & \min_{u_t \in \mathbb{U}} L_t(x, u_t, w) + V_{t+1} \circ f_t(x, u_t, w) \\ V_t(x) & = & \mathbb{E}\left(\hat{V}_t(x, \mathbf{W}_t)\right) \end{cases}$$

Indeed, an optimal policy for this problem is given by

$$\pi_t(x, w) \in \arg\min_{u_t \in \mathbb{U}} \left\{ L_t(x, u_t, w) + V_{t+1} \circ f_t(x, u_t, w) \right\}$$

SDDP : At the beginning of step k

At the beginning of step k, we suppose that we have, for each time step t, an approximation V_t^k of V_t satisfying

- $V_t^k \leq V_t$
- $V_T^k = K$
- V_t^k is convex

SDDP: Forward path: define a trajectory

- Randomly select a scenario $(\xi_0, \dots, \xi_{T-1}) \in \mathbb{W}^T$
- Define a trajectory $(x_t^{(k)})_{t=0,\dots,T}$ by

$$x_{t+1}^{(k)} = f_t(x_t^{(k)}, u_t^{(k)}, \xi_t)$$

where $u_t^{(k)}$ is an optimal solution of

$$\min_{u \in \mathbb{U}} L_t\left(x_t^{(k)}, u, \xi_t\right) + V_{t+1}^{(k)} \circ f_t\left(x_t^{(k)}, u, \xi_t\right)$$

- This trajectory is given by the optimal policy where V_t is replaced by $V_t^{(k)}$

SDDP: Backward path: add cuts

- For any t we want to add a cut to the approximation $V_t^{(k)}$ of V_t
- At time t solve, for any possible w,

$$\hat{\beta}_t^{(k+1)}(w) = \min_{x,u} \quad L_t(x,u,w) + V_{t+1}^{(k+1)} \circ f_t(x,u,w),$$
$$s.t \quad x = x_t^{(k)} \qquad [\hat{\lambda}_t^{(k+1)}(w)]$$

- Compute $\lambda_t^{(k+1)} = \mathbb{E}\left(\lambda_t^{(k+1)}(\mathbf{W}_t)\right)$ and $\beta_t^{(k+1)} = \mathbb{E}\left(\beta_t^{(k+1)}(\mathbf{W}_t)\right)$
- Add a cut

$$V_t^{(k+1)}(x) = \max\left\{V_t^{(k)}(x), \beta_t^{(k+1)} + \left\langle \lambda_t^{(k+1)}, x - x_t^{(k)} \right\rangle\right\}$$

- Go one step back in time: $t \leftarrow t-1$. Upon reaching $t=0$, we have completed step k of the algorithm

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What is StochDynamicProgramming.jl ?

- A package written in Julia, part of JuliaOpt, aimed at solving multistage stochastic optimization.
- The focus is on Dynamic Programming like approaches.
- The idea is to define the SP model:
    - define the dynamic of the system f_t
    - define the costs of the problem L_t
    - define the constraints
    - define noise laws (assumed discrete and time-independent)
- Then it can be solved in three ways:
    - with an extensive formulation approach and calling a solver
    - with a DP approach (defining the discretization of state and controls)
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What is our philosophy ?

- Enable user to easily define the SP model.
- Allow to compare different approaches to tackle a given problem.
- Allow to benchmark variants of SDDP on a given problem.
- Invite academic contributions...

Wait, but is it difficult to install and use ?

Short answer: no.

Long answer:

1. Install Julia (see: http://julialang.org/downloads/).
2. launch julia.
   - Pkg.update()
   - Pkg.add(StochDynamicProgramming)
3. You are all set to go. I suggest looking at stock-example.jl as a tutorial. (notebooks are on the way).

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Conclusion and warning

- This is a free and open-source package for simple implementation of multistage stochastic optimization problem (with an MDP approach).
- It implements three way of solving the problem, with a focus on SDDP.
- It is easy to install and test.
- It is still an early version with lots of evolution to come in the next year...
- But what the package lacks in documentation or stability is compensated by a dedicated support team for our first users ;-)
- Don't hesitate to contact us for help, feedback or contributions !

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