# An adverse selection approach to power tarification

Clemence Alasseur[1]
joint work with Ivar Ekeland [2], Romuald Elie[3], Nicola Hernández Santibáñez[4], Dylan Possomai[2]

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[1] EDF R&D FIME research center
[2] Paris-Dauphine University
[3] Marne la Vallee University
[4] Paris-Dauphine University, University De Chile

# Some elements of electricity retail

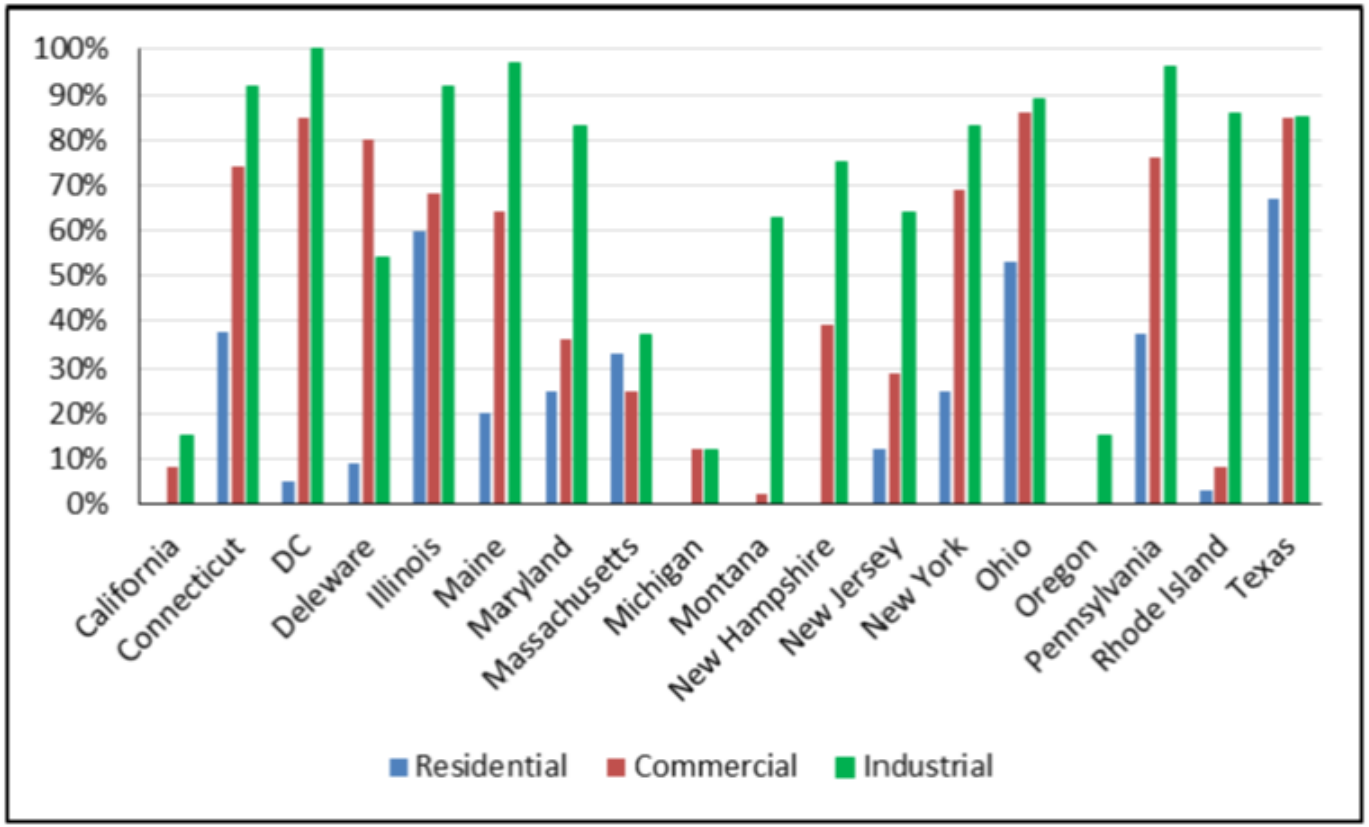

- Competition on electricity retail is a reality!
- New actors are present
- New tariff's structures proposed
  - fixed/variable price
  - duration of engagement
  - source of energy (green...)
  - dual gas/electricity offers
  - standing charges

Example of providers to residential consumers in Bristol, source https://www.ukpower.co.uk

# Some elements of electricity retail

Competition on electricity retail is developing

- Differently against consumer's segment
- Differently against countries




Competitive retail energy supplier's retail sales (share of total MWh) in 2014, source EMRF: "Retail choice in electricity: what have we learned in 20 years"

# Reglemented tarif in France

**Prix de l'électricité TTC au tarif Bleu d'EDF selon la puissance et l'option tarifaire à jour au 8 novembre 2016**

| Puissance souscrite | Option Base | | Option Heures Pleines - Heures Creuses | | |
|---|---|---|---|---|---|
| | Abonnement annuel | Prix du kWh | Abonnement annuel | Prix du kWh heures pleines | Prix du kWh heures creuses |
| 3 kVA | 56.07 € | 0.15640 € | | | |
| 6 kVA | 96.50 € | 0.14490 € | 100.51 € | 0.15600 € | 0.12700 € |
| 9 kVA | 111.35 € | 0.14620 € | 117.50 € | 0.15600 € | 0.12700 € |
| 12 kVA | 172.78 € | 0.14620 € | 183.25 € | 0.15600 € | 0.12700 € |
| 15 kVA | 199.59 € | 0.14620 € | 212.05 € | 0.15600 € | 0.12700 € |
| 18 kVA | 228.56 € | 0.14620 € | 239.84 € | 0.15600 € | 0.12700 € |
| 24 kVA | 491.85 € | 0.14620 € | 495.65 € | 0.15600 € | 0.12700 € |
| 30 kVA | 594.30 € | 0.14620 € | 599.74 € | 0.15600 € | 0.12700 € |
| 36 kVA | 698.64 € | 0.14620 € | 703.07 € | 0.15600 € | 0.12700 € |

Figure: Reglemented electricity tarif

# Motivations

- Which tariff's structure should propose electricity providers?
  - Depending on level of competition against electricity providers?
  - Depending on power plants used to generate electricity?
  - To enable risk sharing between providers and consumers? ...

- Considering
  - Electricity is a staple good (difficult to substitute)
  - Tariff's structure should remain simple

In this work, not every questions are solved! We propose a mathematical framework simple enough so that we can explicitly solve the problem.

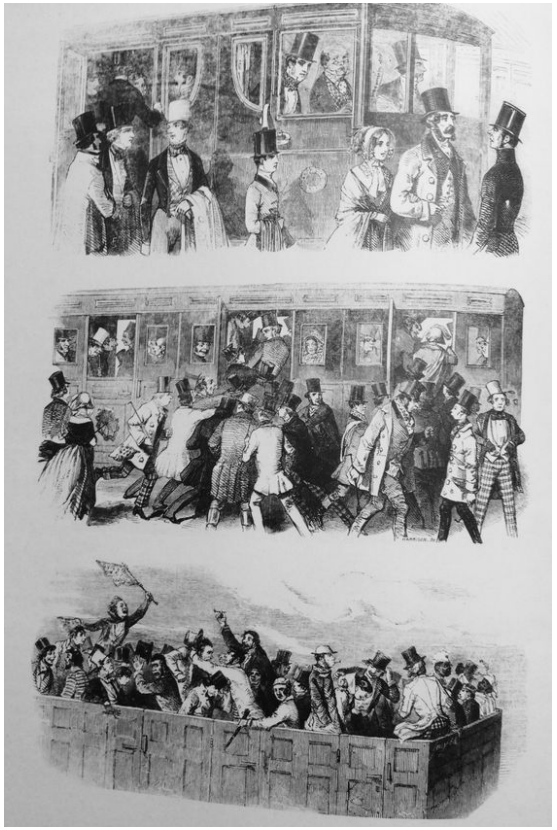
# Principal-Agent model

2 types of actors :

- The Principal: proposes the contract
- The Agent: accepts or rejects the contract

Information can be incomplete for one of these two parties.

- Moral hazard : the Principal only observes the outcome and not directly the action of the agent
- **Adverse selection**: Agent's characteristic is imperfectly observed by the Principal.The Principal offers the agent a menu of contracts designed such that the Agent reveals its characteristic.
  Some examples:
  - Trains: different classes to distinguish passengers'willingness to pay
  - Insurance: deductible to distinguish drivers's nature



# Objectives of actors

Each Agent has its own willingness to consume electricity depending on its type. Its type is unknown to the Principal but the Principal knows the repartition of Agent's type.

**Objective of the Agent**: to select the consumption level which maximize its utility minus the tariff he needs to pay for the consumed electricity.

**Objective of the Principal**: to propose a tariff which maximize its own profits = payments received by consumers who take its contract minus costs for producing/providing the electricity consumption of ALL its clients.

**2 conditions**

- Individual Rationality: one Agent accepts the contract only if its benefit to accept it is higher than its reservation utility.
- Incentive Compatibility : each Agent prefers the contract that was designed for his particular type.

# Electricity production

Originality: electricity marginal price increases with the total consumption!

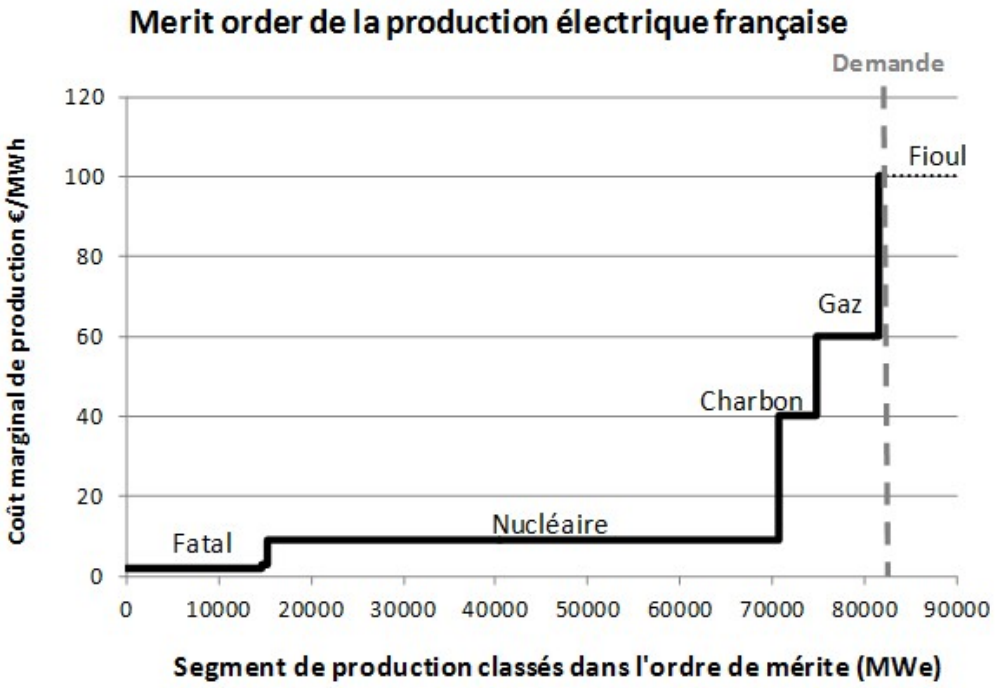



Figure: example of merit order of French electricity production, source http://conseils.xpair.com/actualite_experts/valeur-contenu-co2-electricite.htm

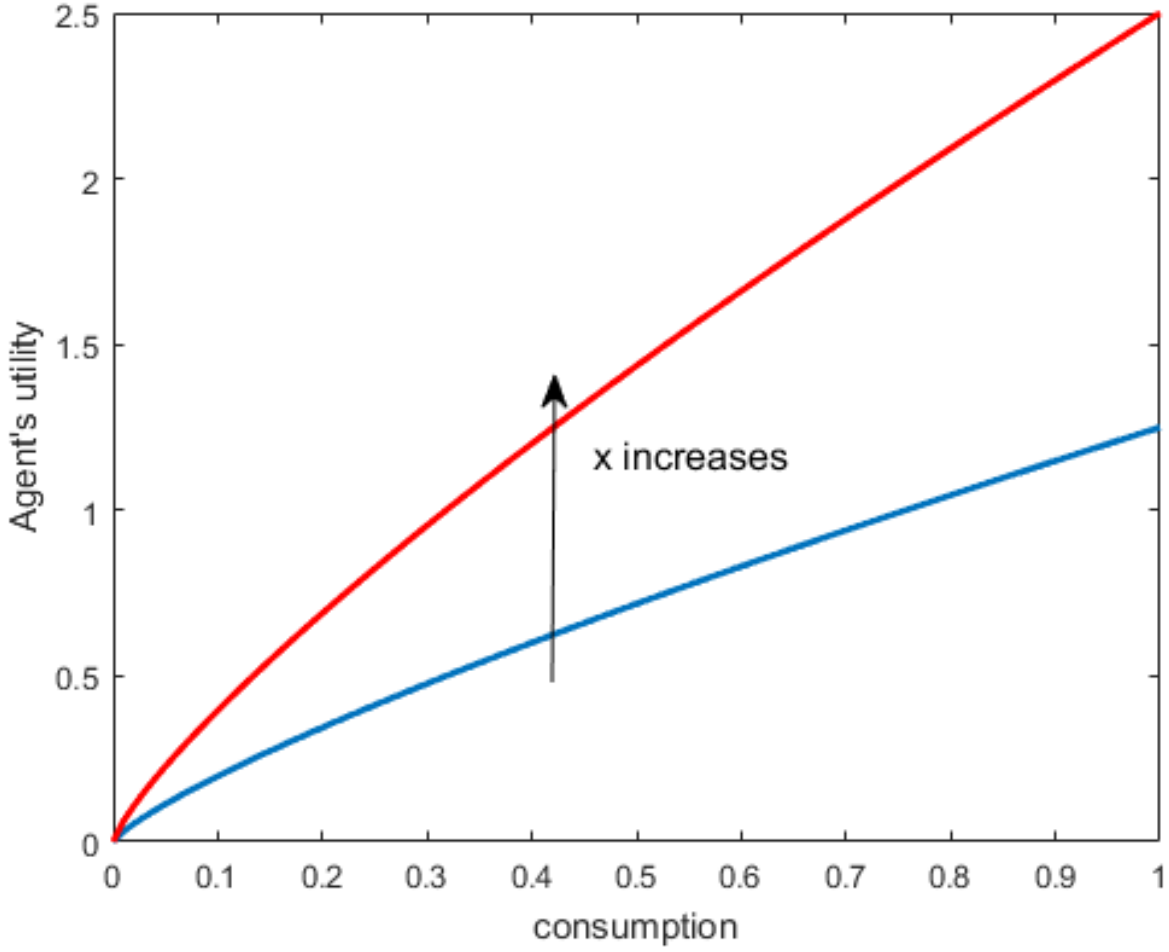
# Notations and assumptions

- $p : [0, T] \times \mathcal{C} \longrightarrow \mathbb{R}_+$ the tariff proposed by the Principal, $p(t, c)$ the price of an amount $c$ of electricity at time $t$ (we restrict to only one contract and not a menu).
- $K : [0, T] \times \mathcal{C} \longrightarrow \mathbb{R}_+$ is the cost of production of electricity for the Principal:
$$K(t, c) = \frac{k(t)}{n} c^n$$
with $k(t) \in \mathbb{R}_+^*$ and $n > 1$
- Electricity consumption $c \in \mathcal{C} = \mathbb{R}_+$ or $\mathbb{R}_+^*$ and $x \in [0, 1]$ the type of the agent.
- Uniform repartition of agents' type $f(x) = 1$
- CRRA utility (constant relative risk aversion) of the agent:
$$u(t, c, x) = g_\gamma(x)\phi(t)\frac{c^\gamma}{\gamma}$$
  - $\phi(t)$ eagerness to consume depending on time
  - $g_\gamma(x)$ willingness of the Agents to pay for $c$ and we study $g_\gamma(x) := x\mathbf{1}_{\gamma \in (0,1)} + (1 - x)\mathbf{1}_{\gamma < 0}$
- Reservation utility $h(t, x)$

# First case - $0 < \gamma < 1$ et $g(x) = x$

Electricity is a market product and people can substitute it

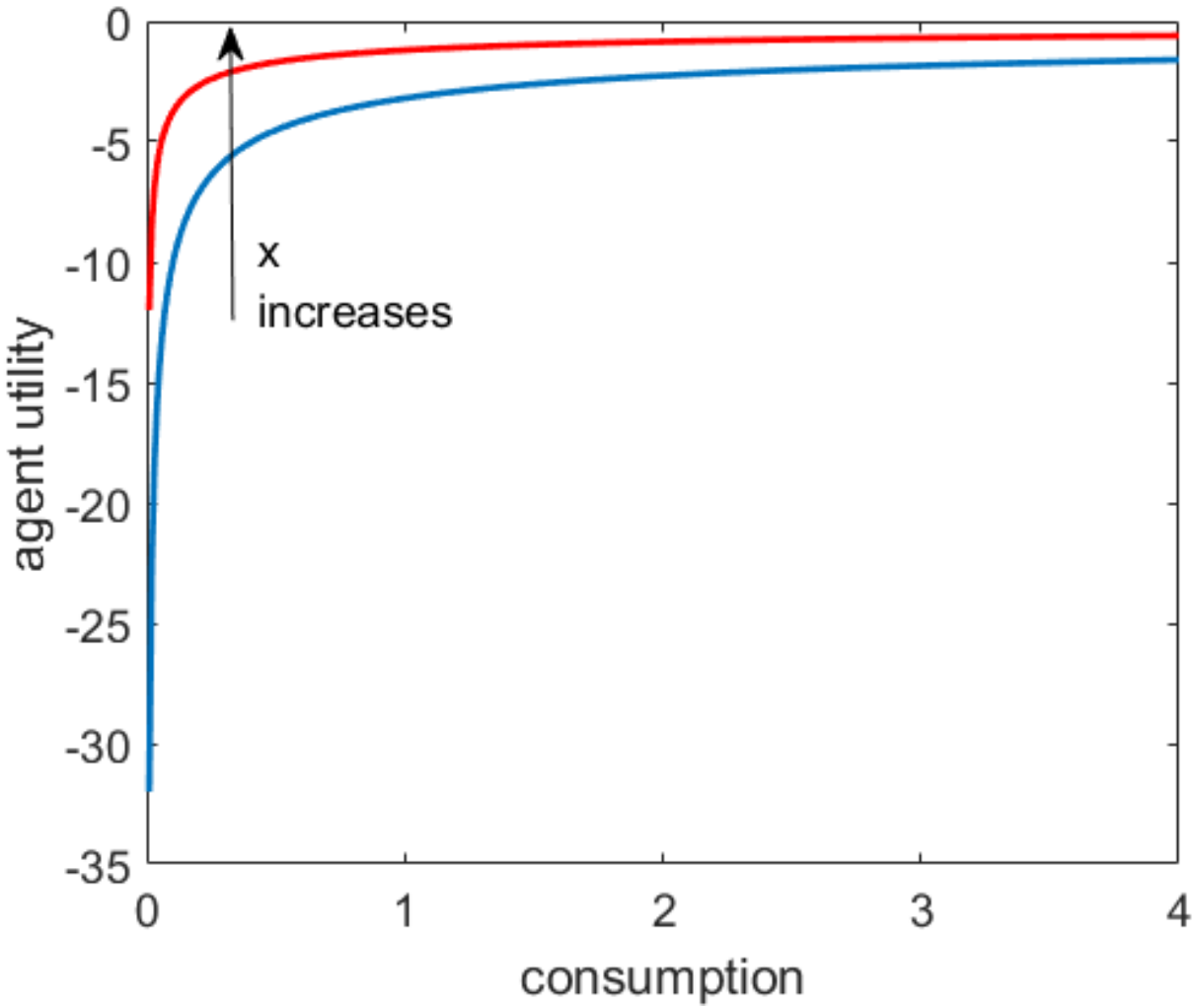
$$u(t, c, x) = x\phi(t)\frac{c^\gamma}{\gamma}$$



# Second case - $\gamma < 0$ et $g(x) = 1 - x$

Electricity is a staple product and it is unconceivable not to consume it.

$$u(t, c, x) = (1 - x)\phi(t)\frac{c^\gamma}{\gamma}$$



# Maximum welfare and admissible contract

Let's define $p^\star : [0, T] \times X \longrightarrow \mathbb{R}$ the maximum welfare (utility minus payment) the consumer can achieve with tariff $p$:

$$p^\star(t, x) := \sup_{c \in \mathcal{C}} \{u(t, x, c) - p(t, c)\}, \text{ for any } (t, x) \in [0, T] \times X.$$

Definition

A tariff $p : [0, T] \times \mathbb{R}_+ \longrightarrow \mathbb{R}$ will be said to be admissible, denoted by $p \in \mathcal{P}$, if

- The maximum welfare is attainable: it exists at least one $c^\star$ such that $p^\star(t, x) = u(t, x, c^\star) - p(t, c^\star)$
- The tariff can be derived from $p^\star$: $p(t, c) = \sup_{x \in X} \{u(t, x, c) - p^\star(t, x)\}$.

# Agent's problem

The Agent of type $x \in X$ determines his consumption by solving the following problem

$$U_A(p, x) := \sup_{c \in \mathfrak{C}} \int_0^T (u(t, x, c(t)) - p(t, c(t)))\, dt. \tag{1}$$

Proposition

*For every $p \in$ and for almost every $x \in X$, we have*

$$U_A(p, x) = \int_0^T p^\star(t, x) dt,$$

*and the optimal consumption of Agents of type $x$ at any time $t \in [0, T]$ is given by*

$$c^\star(t, x) = \left(\frac{\partial u}{\partial x}(t, x, \cdot)\right)^{(-1)} \left(\frac{\partial p^\star}{\partial x}(t, x)\right) = \left(\frac{\gamma}{\phi(t)\, g'_\gamma(x)} \frac{\partial p^\star}{\partial x}(t, x)\right)^{\frac{1}{\gamma}} \tag{2}$$

# Principal's problem

Proposed tariff should satisfy two conditions:

- Incentive Compatibility (IC): automatically satisfied as no moral hazard and only one contract
- Individual Rationality (IR)

The set $X^\star$ of the types of Agents which accept the contract:

$$X^\star(p^\star) := \left\{ x \in X,\ P^\star(x) := \int_0^T p^\star(t,x)dt \geq \int_0^T h(t,x)dt =: H(x) \right\},$$

The Principal sets a tariff $p \in$ as a solution to his maximization problem

$$U_P := \sup_{p \in P} \int_0^T \left[ \int_{X^\star(p^\star)} p(t, c^\star(t,x)) f(x)dx - K\left(t, \int_{X^\star(p^\star)} c^\star(t,x) f(x)dx\right) \right] dt. \tag{3}$$

This problem can be rewritten in terms of $p^\star$ only as

$$U_P = \sup_{p \in P} \int_0^T \left[ \int_{X^\star(p^\star)} \left( \frac{g_\gamma(x)}{g'_\gamma(x)} \frac{\partial p^\star}{\partial x}(t,x) - p^\star(t,x) \right) dx \right.$$
$$\left. - K\left(t, \int_{X^\star(p^\star)} \left( \frac{\gamma}{\phi(t) g'_\gamma(x)} \frac{\partial p^\star}{\partial x}(t,x) \right)^{\frac{1}{\gamma}} dx \right) \right] dt. \tag{4}$$

# Our problem

Important remarks:

- NO a priori on the tariff's structure!
- the cost K is function of consumption of every consumers.

Our problem: find an admissible tariff such that:

$$X^\star(p^\star) := \left\{ x \in X,\ U_A(p,x) = \int_0^T p^\star(t,x)dt \geq \int_0^T h(t,x)dt =: H(x) \right\}$$

$$U_P = \sup_{p \in P} \int_0^T \left[ \int_{X^\star(p^\star)} \left( \frac{g_\gamma(x)}{g'_\gamma(x)} \frac{\partial p^\star}{\partial x}(t,x) - p^\star(t,x) \right) dx \right.$$
$$\left. - K\left( t, \int_{X^\star(p^\star)} \left( \frac{\gamma}{\phi(t) g'_\gamma(x)} \frac{\partial p^\star}{\partial x}(t,x) \right)^{\frac{1}{\gamma}} dx \right) \right] dt. \qquad (5)$$

# Sketch of the proof

Let's consider space $C^+$ of maps $g$ such that for every $t$, $x \longmapsto g(t,x)$ is continuous and non-decreasing.

1 - Consider the alternative problem $\tilde{U}_P = \sup_{p^* \in C^+} \ldots \geq U_P = \sup_{p \in P} \ldots$

2 - Compute $\tilde{U}_P$

a) Prove $X^*$ is the union of one or two intervals

b) Integrate by part: $\tilde{U}_P = \sup_{x_0} \sup_{p^* \in C^+} \psi_{x_0}(p^*)$ with $\psi_{x_0}$ concave

c) $\psi_{x_0}$ Frechet differentiable, $\psi'_{x_0} \leq 0$ in every direction

d) Get $\frac{\partial p^\star}{\partial x}(t,x) = f(t,x_0)$

e) Inject $\frac{\partial p^\star}{\partial x}$ into $\tilde{U}_P$ enables to find $x_0$

3 - Prove the two problems are equal: $p^*$ is $u-$convex and then $p$ is solution of initial problem

# Constant reservation utility $h(t,x) = h(t)$

Theorem

*Only the Agents of type* $x \geq x^\star_{0,\gamma}$ *will accept the contract:* $X^\star(p^\star) = [x^\star_{0,\gamma}, 1]$

$x^\star_{0,\gamma}$ *is unique*

*The optimal tariff* $p \in \mathcal{P}$ *is the following:*
*(i) If* $\gamma \in (0,1)$

$$p(t,c) = \begin{cases} p_\gamma(t)c^\gamma + p^{>}_{\gamma,\,const}(t), & \text{if } c > c^*_\gamma(t) \\ \dfrac{p_\gamma(t)}{2}c^\gamma + p^{<}_{\gamma,\,prop}(t)c + p^{<}_{\gamma,\,const}(t), & \text{otherwise,} \end{cases}$$

*and* $x^\star_0$ *is in* $(1/2, 1)$

*(ii) If* $\gamma < 0$

$$p(t,c) = \begin{cases} p_\gamma(t)c^\gamma a + p^{>}_{\gamma,\,const}(t), & \text{if } c > c^*_\gamma(t) \\ p^{<}_{\gamma,\,prop}(t)c + p^{<}_{\gamma,\,const}(t), & \text{otherwise,} \end{cases}$$

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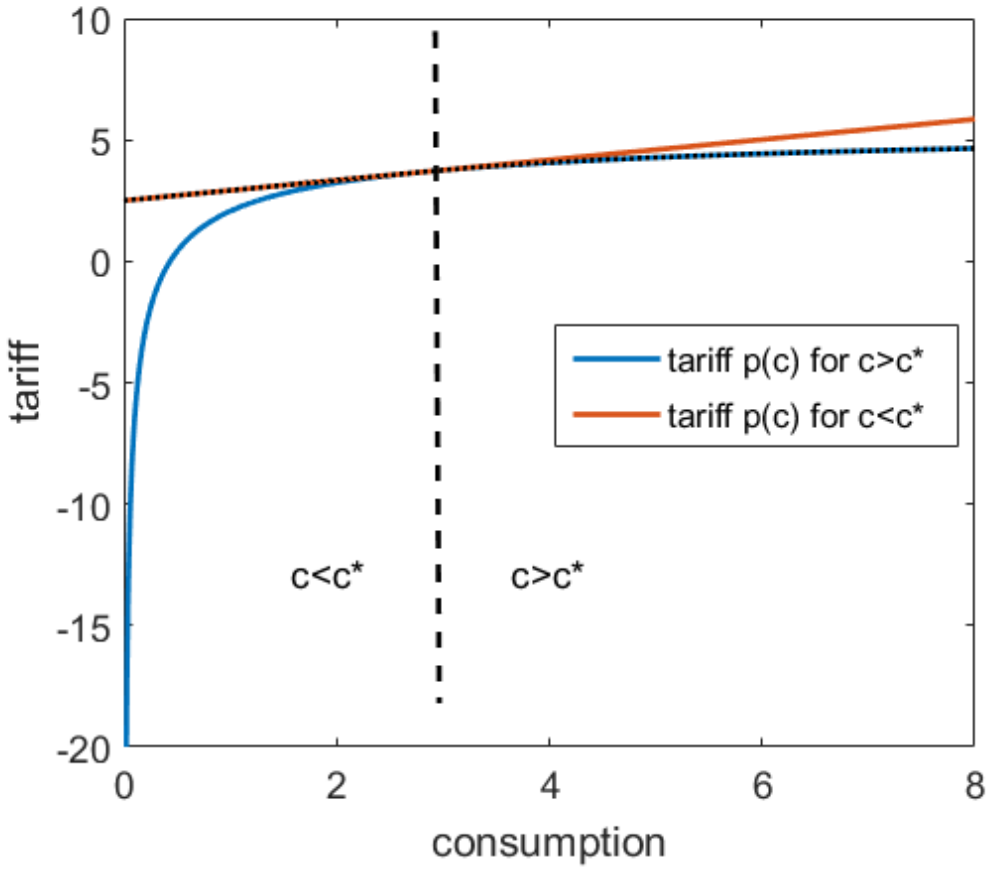
$$p(t,c) = \begin{cases} p_\gamma(t)c^\gamma a + p^{>}_{\gamma,\,const}(t), & \text{if } c > c^*_\gamma(t) \\ p^{<}_{\gamma,\,prop}(t)c + p^{<}_{\gamma,\,const}(t), & \text{otherwise,} \end{cases}$$

*and* $x^\star_0$ *is in* $(0,1)$

Consumers who select the Principal chose to consume less electricity than $\hat{c}^*_\gamma$

# Illustration of tariff - $h(t,x) = h(t)$, $\gamma < 0$ and $g(x) = 1 - x$




Remark: to propose following tariff $p(t, c) = p^{<}_{\gamma,\,\text{prop}}(t)c + p^{<}_{\gamma,\,\text{const}}(t)$ is equivalent to propose optimal tariff $p \in \mathcal{P}$:

$$p(t, c) = \begin{cases} p_\gamma(t)c^\gamma a + p^{>}_{\gamma,\,\text{const}}(t), & \text{if } c > c^*_\gamma(t) \\ p^{<}_{\gamma,\,\text{prop}}(t)c + p^{<}_{\gamma,\,\text{const}}(t), & \text{otherwise,} \end{cases}$$

# Some results - $h(t,x) = h(t)$

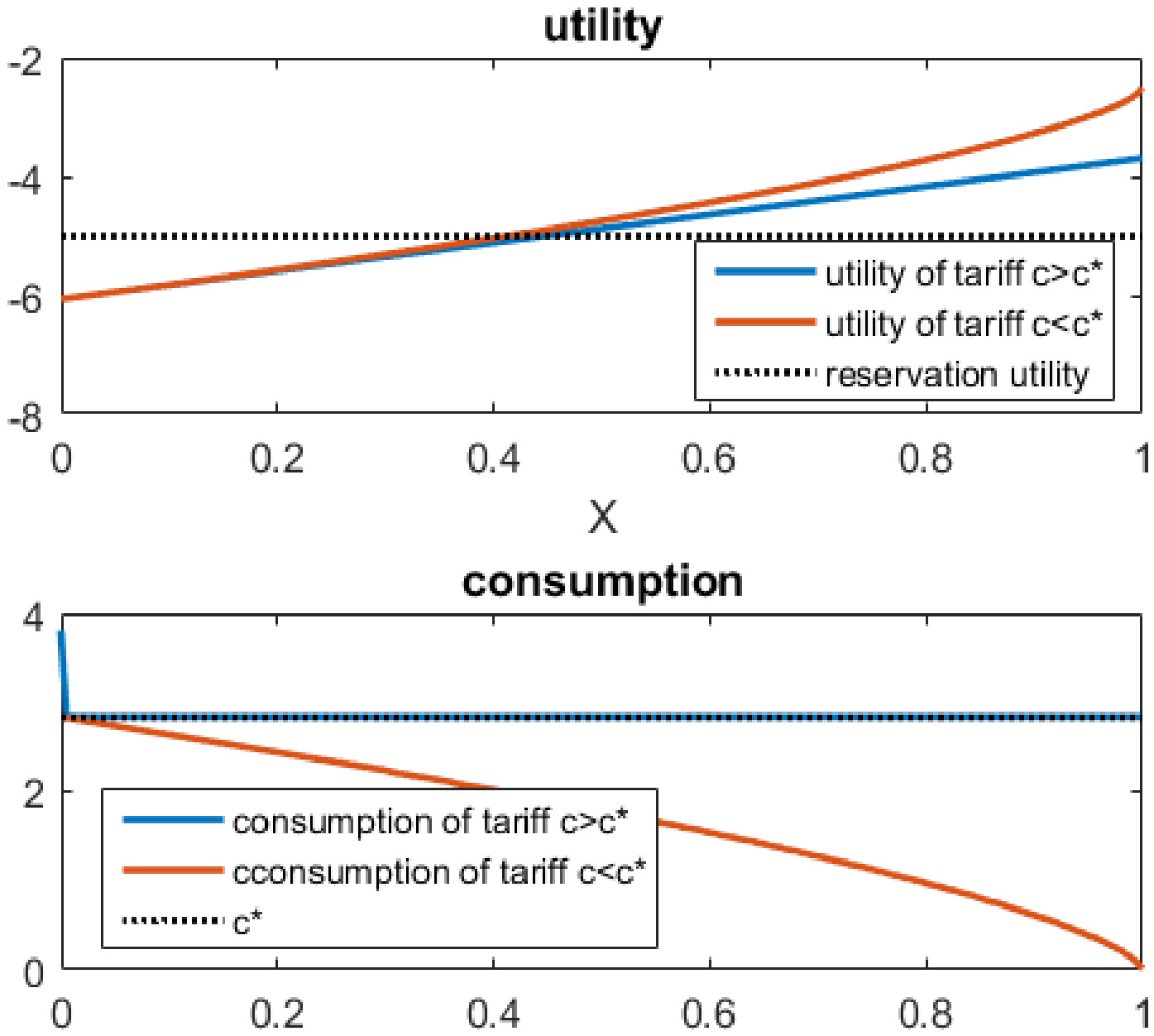
- The structure of the proposed contract is very simple:

$$p(t,c) = p_{const}(t) + p_{prop}(t).c + p_{\gamma}(t).c^{\gamma}.1_{0<\gamma<1}$$

- $p_{\gamma}(t)$ is independent of the Principal's characteristics, only depends on Agent's utility

- The Principal signs contract only with Agents of type $x \in [x_0, 1]$ - when $\gamma \in [0,1]$ corresponds to Agents with highest consumption's level
  - when $\gamma < 0$ corresponds to Agents with lowest consumption's level
- Classical result of informational rent: the more efficient Agents get a tariff inferior to what they are eager to pay whereas the less efficient Agents pay all what they are able to pay or are excluded.

# Some results - $h(t, x) = h(t)$, $\gamma < 0$



# Some results - $h(t,x) = h(t)$, continued

$$p(t,c) = p_{const}(t) + p_{prop}(t).c + p_{\gamma}(t).c^{\gamma}.1_{0<\gamma<1}$$

- Impact of production competition: when $h$ increases
  - The Principal has less clients ($x_0$ increases)
  - The Principal decreases $p_{const}(t)$
  - Remaining clients consume more

- Impact of production costs: when $k$ increases
  - The Principal has less clients ($x_0$ increases)
  - The Principal increases $p_{prop}(t)$
  - Remaining clients consume less
  - $p_{prop}(t)$ decreases when $n$ increases (ie higher convexity of the production cost function)

# Some results - $h(t,x) = h(t)$, $\gamma < 0$, continued

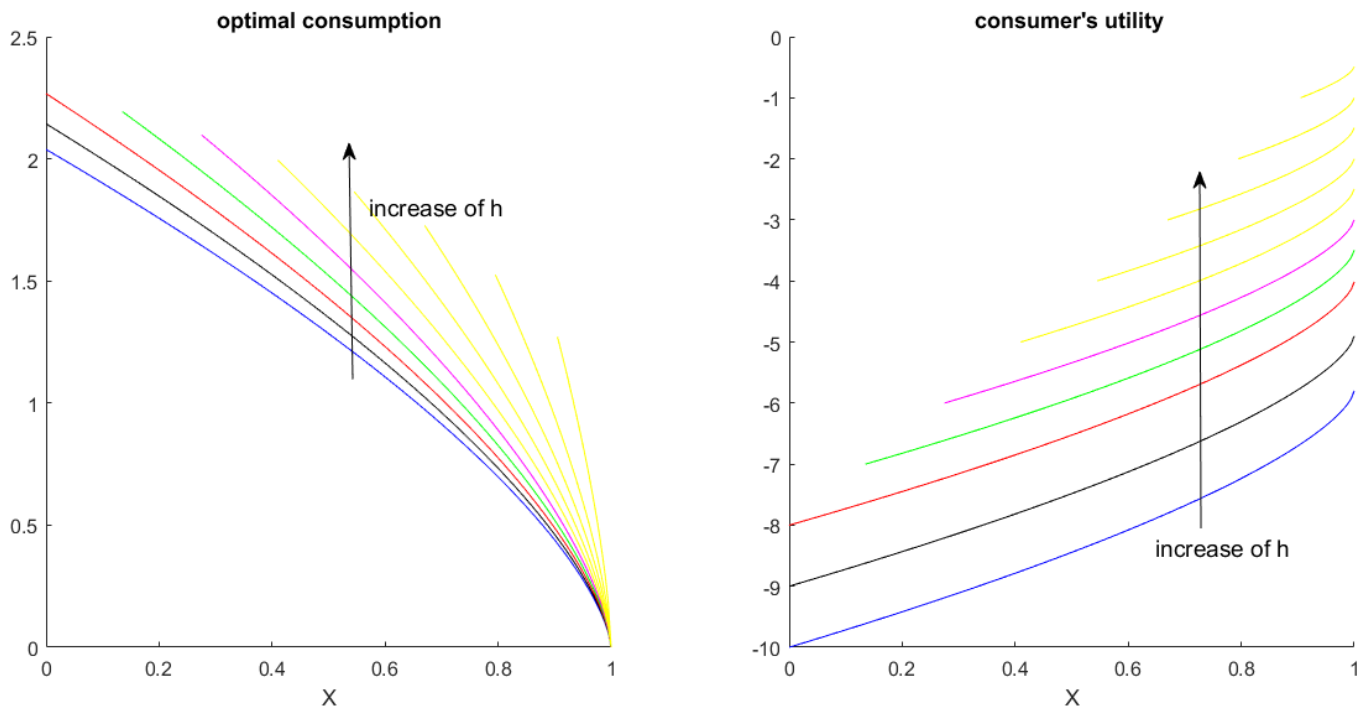



Figure: utility and consumption evolution when h increases

# Some results - $h(t, x) = h(t)$, $\gamma < 0$, continued

$$p(t, c) = p_{const}(t) + p_{prop}(t).c$$

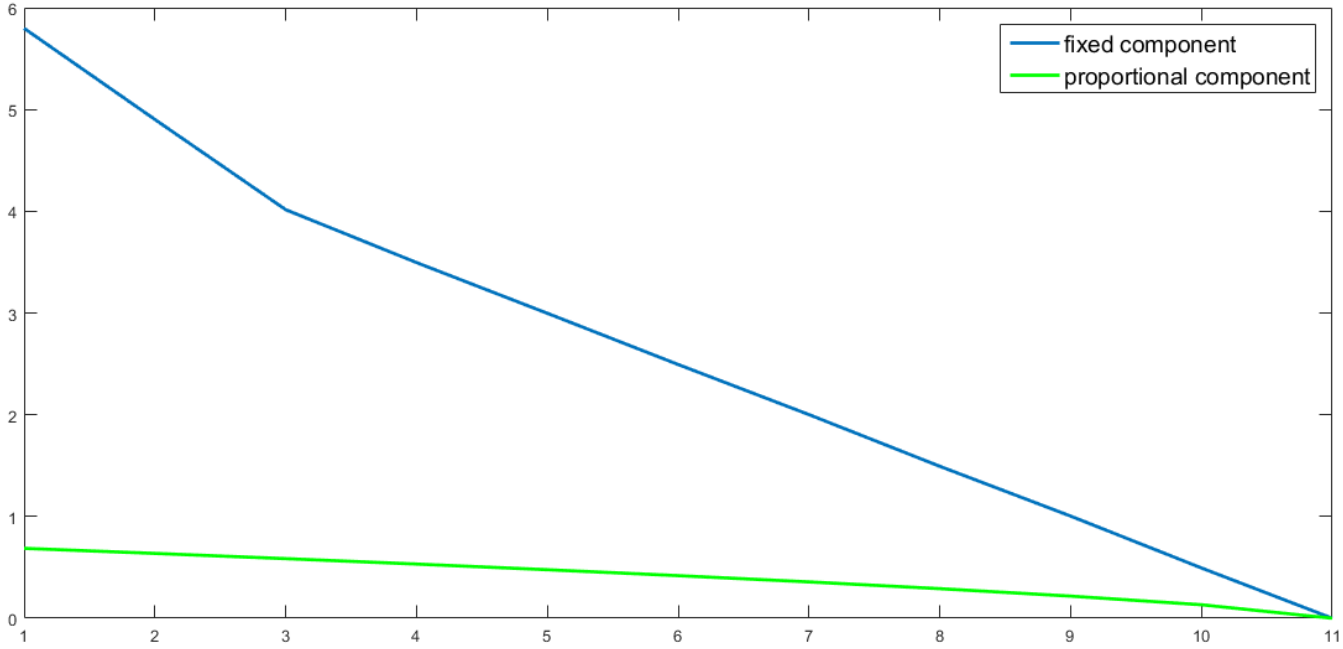



Figure: Evolution of tariff's component with h

In this example, the standing charge $p_{const}$ represents a big amount of the payment compared to sum of charges per unit consumed.

# Some results - concave $h(t, x)$

- If $X \longleftarrow H(x$ is )strictly concave and non decreasing, only Agents of type $X^\star(p^\star) = [0, x_1] \cup [x_2, 1]$ accept the contract.
- This is an original result of informational rent
- The structure of proposed contract remains similar:

$$p(t, c) = p^i_{const}(t) + p^i_{prop}(t).c + p^i_\gamma(t).c^\gamma \text{if } c \in [c^{*,i-1}, c^{*,i}]$$

with four different regions $i$

# Conclusions and future extensions

In the proposed framework: explicit tariffs are provided and those tariffs are simple functions of consumption.

$$p(t, c) = p_{const}(t) + p_{prop}(t).c + p_{\gamma}(t).c^{\gamma}$$

Future extensions we plan to study:

- To include additional Agents'characteristics such as their taste for green contracts, how fast they are changing retailers with respect to price difference...
- To take into account a maximum instantaneous consumption in tariff
- To take into account explicitly the maximum production level of the system
- To introduce uncertainty on consumption and production

Some references:


- P. Joskow, J. Tirole, Retail electricity competition, RAND Journal of Economics, 2006
- B. Salanie, The Economics of Contracts, MIT Press

# Constant reservation utility $h(t,x) = h(t)$ and $\gamma \in [0,1]$

only the Agents of type $x \geq \widehat{x}_0^\star$ will accept the contract: $X^\star(p^\star) = [x_0, 1]$

Theorem

*$(i)$ If $\gamma \in (0,1)$, then, the optimal tariff $p \in \mathcal{P}$*

$$p(t,c) = \begin{cases} \phi(t)\dfrac{c^\gamma}{\gamma} + M(t)\left((2x_0^\star - 1)^{\frac{1}{1-\gamma}} - 1\right) - h(t), \ \textit{if } c > \left(\dfrac{2\gamma M(t)}{(1-\gamma)\phi(t)}\right)^{\frac{1}{\gamma}}, \\ \phi(t)\dfrac{c^\gamma}{2\gamma} + \left(\left(\dfrac{\phi(t)}{2}\right)^{\frac{1}{1-\gamma}} \dfrac{1-\gamma}{\gamma M(t)}\right)^{\frac{1-\gamma}{\gamma}} c - h(t) + M(t)(2x_0^\star - 1)^{\frac{1}{1-\gamma}}, \ \textit{other} \end{cases}$$

*where*

$$M(t) = \frac{1-\gamma}{2\gamma}\left(\frac{2(2-\gamma)}{1-\gamma}\right)^{\frac{\gamma(n-1)}{n-\gamma}} \left(\frac{\phi^n(t)}{k^\gamma(t)}\right)^{\frac{1}{n-\gamma}} \left(1 - (2x_0^\star - 1)^{\frac{2-\gamma}{1-\gamma}}\right)^{-\frac{\gamma(n-1)}{n-\gamma}},$$

*and where $x_0^\star$ is the unique solution in $(1/2, 1)$ of the equation*

$$\int_0^T h(t)dt = 2nA_\gamma(T)\frac{2-\gamma}{n-\gamma}(2x_0^\star - 1)^{\frac{1}{1-\gamma}} \left(1 - (2x_0^\star - 1)^{\frac{2-\gamma}{1-\gamma}}\right)^{-\frac{\gamma(n-1)}{n-\gamma}}.$$

# Constant reservation utility $h(t,x) = h(t)$ and $\gamma < 0$

only the Agents of type $x \geq \widehat{x}_0^\star$ will accept the contract: $X^\star(p^\star) = [x_0, 1]$

Theorem

*The optimal tariff $p \in \mathcal{P}$*

$$p(t,c) = \begin{cases} \phi(t)\dfrac{c^\gamma}{\gamma} - h(t) - \widehat{M}(t)(1-\widehat{x}_0^\star)^{\frac{1}{1-\gamma}} + \widehat{M}(t), \ \textit{if } c > \left(-\dfrac{\gamma\widehat{M}(t)}{\phi(t)(1-\gamma)}\right)^{\frac{1}{\gamma}}, \\ -\gamma c\left(-\dfrac{\phi(t)}{\gamma}\right)^{\frac{1}{\gamma}}\left(\dfrac{1-\gamma}{\widehat{M}(t)}\right)^{\frac{1-\gamma}{\gamma}} - h(t) - \widehat{M}(t)(1-\widehat{x}_0^\star)^{\frac{1}{1-\gamma}}, \ \textit{otherwise.} \end{cases}$$

*where*

$$\widehat{M}(t) = -\frac{1-\gamma}{\gamma}\left(\frac{2-\gamma}{1-\gamma}\right)^{\frac{\gamma(n-1)}{n-\gamma}}\left(\frac{2^\gamma\phi^n(t)}{k^\gamma(t)}\right)^{\frac{1}{n-\gamma}}(1-\widehat{x}_0^\star)^{-\frac{\gamma(2-\gamma)(n-1)}{(n-\gamma)(1-\gamma)}},$$

*and where*

$$\widehat{x}_0^\star := \left(1 - \left(\frac{n-\gamma}{n(1-\gamma)B_\gamma(T)}\int_0^T h(t)dt\right)^{\frac{n-\gamma}{n(1-\gamma)+\gamma}}\left(\frac{2-\gamma}{1-\gamma}\right)^{\frac{-\gamma(n-1)}{n(1-\gamma)+\gamma}} 2^{\frac{-n}{n(1-\gamma)+\gamma}}\right)^+.$$

# Concave reservation utility and $\gamma < 0$

$$
\text{p(t,c)}=\begin{cases}
\phi(t)\dfrac{c^{\gamma}}{\gamma} - N_{\gamma}\left(2^{-\frac{1}{1-\gamma}} - \left(a_0^{\star} - \dfrac{1}{2}\right)^{\frac{1}{1-\gamma}}\right) - h(t, a_0^{\star}), \text{ if } c > L_{\gamma}(t)2^{-\frac{1}{1-\gamma}}, \\
\phi(t)\dfrac{c^{\gamma}}{2\gamma} + \phi(t)L_{\gamma}(t)^{\gamma-1}c + N_{\gamma}\left(a_0^{\star} - \dfrac{1}{2}\right)^{\frac{1}{1-\gamma}} - h(t, a_0^{\star}), \text{ if } L_{\gamma}(t)\left(a_0^{\star} - \dfrac{1}{2}\right)^{\frac{1}{1-} \\
\tilde{x}^{\star}(c)\phi(t)\dfrac{c^{\gamma}}{\gamma} - \tilde{p}^{\star}(t, \tilde{x}(c)), \text{ if } L_{\gamma}(t)(b_0^{\star})^{\frac{1}{1-\gamma}} < c \leq L_{\gamma}(t)\left(a_0^{\star} - \dfrac{1}{2}\right)^{\frac{1}{1-\gamma}}, \\
\phi(t)L_{\gamma}(t)^{\gamma-1}c - h(t, b_0^{\star}) + N(b_0^{\star})^{\frac{1}{1-\gamma}}, \text{ if } 0 \leq c \leq L_{\gamma}(t)(b_0^{\star})^{\frac{1}{1-\gamma}}.
\end{cases}
$$