Nash Stability in Hedonic Skill Games

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POC - October 23

Outline

Coalitional Formation Games

A set of players $\mathcal{N}$ (player=agent)

A state of the game is a **coalition structure**, i.e., partition of $\mathcal{N}$

Example

$\mathcal{N} = \{$Alice, Bob, Cathy, Dylan, Eric, Fred, Greg, Herbert$\}$

Coalition structure: {Alice, Bob, Herbert} {Cathy, Dylan, Fred, Greg} {Eric}

Every player has some preference relation over the set of all possible coalition structures

Hedonic case: The preference of a player only depends on her coalition

Ranking of $2^{|\mathcal{N}|-1}$ possible coalitions for every player $\Rightarrow$ difficult to succinctly represent the game

Solution concepts

Contractual individual stability

A partition is said to be **contractually individually stable** if no player can benefit from moving from her coalition S to another coalition T (T may be empty) while not making the members of $S \cup T$ worse off.

Individual stability

A partition is said to be **individually stable** if no player can benefit from moving from her coalition S to another coalition T (T may be empty) while not making the members of T worse off.

Nash stability *

A partition is said to be **Nash stable** if no player can benefit from moving from her coalition S to another coalition T.

Hedonic Skill Games

A special coalitional game admitting a **succinct representation**

The game uses **utilities** instead of preferences: a player prefers coalition C over coalition C' if her utility for C is larger than her utility for C'

Coalitions can perform some **weighted tasks** requiring **skills**

Skills are possessed by the players

The utility of a player depends on the weight of the tasks that her coalition can perform

Applications: research teams, volunteers in charity organizations, rescue squads, political parties, . . .

Hedonic Skill Games: formal definition

A set of players (a.k.a. agents) $\mathcal{N} = \{1, \ldots, n\}$, a set of tasks $\mathcal{T} = \{t_1, \ldots, t_m\}$, and a set of skills $\mathcal{S} = \{s_1, \ldots, s_k\}$

Each player ℓ possesses a set of skills $S(\ell) \subseteq \mathcal{S}$, and each task t_j requires a set of skills $S(t_j) \subseteq \mathcal{S}$

Each tasks t_j has a weight $w(t_j) \in \mathbb{R}_{\geq 0}$

Each state is a coalition structure (i.e., partition of $\mathcal{N}$)

The skills of a coalition C is $S(C) = \bigcup_{\ell \in C} S(\ell)$

A coalition C can perform a task t_j iff $S(C) \supseteq S(t_j)$

$T(C)$ = set of tasks performed by coalition C

Example

	s_1	s_2	s_3
t_1	✓	✓	
t_2		✓	✓

	weight
t_1	6
t_2	4

	s_1	s_2	s_3
P1	✓	✓	
P2		✓	✓
P3			✓

$\{P1, P2, P3\}$: the three agents are in the same grand coalition C, both tasks are performed by C

$\{P1, P3\}\{P2\}$: both tasks are performed in coalition $\{P1, P3\}$; only task t_2 is performed in $\{P2\}$

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Utility of the agents

An agent's utility depends on the weight of the tasks that she is participating in

The total weight of the tasks performed in coalition C is distributed over the agents of C

The utility of agent $\ell \in C$ (to be maximized) is equal to

$$\sum_{t \in T(C)} \sum_{s \in S(t) \cap S(\ell)} \frac{w(t)}{|S(t)| \cdot |\{a \in C \mid s \in S(a)\}|}$$

where $T(C)$ is the set of tasks performed by coalition C

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Utility of the agents (example)

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t_2		✓	✓

	weight
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	s_1	s_2	s_3
P1	✓	✓	
P2		✓	✓
P3			✓

Suppose the coalition structure is $\{P1, P3\}\{P2\}$, where both tasks are performed in $\{P1, P3\}$ whereas only t_2 is performed by $\{P2\}$

Utility of P1=6+2=8 (total weight of t_1+ half the weight of t_2)

Utility of P2=4 (total weight of t_2)

Utility of P3=2 (half the weight of t_2)

Hedonic Skill Games

Each coalition performs a task at most once

A task can be performed by more than one coalition

Parameter q: maximum number of coalitions in a coalition structure

Social welfare

Sum of the players' utilities = total weight of the performed tasks

	s_1	s_2	s_3
t_1	✓	✓	
t_2		✓	✓

	weight
t_1	6
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	s_1	s_2	s_3
P1	✓	✓	
P2		✓	✓
P3			✓

$\{P1, P3\}\{P2\}$: social welfare $=6+4+4=14$

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t_2	4

	s_1	s_2	s_3
P1	✓	✓	
P2		✓	✓
P3			✓

$\{P1, P3\}\{P2\}$: social welfare =6+4+4=14

Questions about the hedonic skill game

Can we guarantee the existence of a Nash stable outcome?

How difficult is the computation of a Nash stable outcome?

How difficult is the computation of a social optimum?

Does a natural dynamics converge?

How bad is a Nash stable outcome w.r.t. the social welfare?

This talk: answers for $q \in \{2, n\}$, i.e., at most 2 coalitions or any number of coalitions

Bad news

	s_1	s_2
t_1	✓	✓

	weight
t_1	2

	s_1	s_2
P1	✓	✓
P2		✓

$$\begin{array}{ccccc} \{P1, P2\} & \rightarrow & \{P1\}\{P2\} & \rightarrow & \{P1, P2\} \\ (\frac{3}{2}, \frac{1}{2}) & \rightarrow & (2, 0) & \rightarrow & (\frac{3}{2}, \frac{1}{2}) \end{array}$$

If the two players are together, then $P1$ prefers to be alone

If the two players are separated, then $P2$ prefers to be in the same coalition as $P1$

No state of this instance is Nash stable

Characterizing instances admitting a Nash stable outcome

	s_1	s_2
t_1	✓	✓

	weight
t_1	2

	s_1	s_2
P1	✓	✓
P2		✓

In the bad example, one player has more than one skill, and one task requires more than one skill

Singleton agents instances: every agent has exactly one skill

Singleton tasks instances: every task requires exactly one skill

A Nash stable always exists in both *singleton* cases

Singleton agents instances

Hypothesis: every agent has exactly one skill, but a task can require more than one skill

Better Response dynamics (BRD)

Start from any pure strategy state. If a player has a profitable deviation (better response), then do the move

BRD eventually converges to a Nash stable outcome if the game admits a **potential**

Potential: a real associated with each state such that every profitable deviation induces an increase of the potential

Does BRD always ends on a local optimum of some potential for the hedonic skill game with singleton agents?

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Does BRD always ends on a local optimum of some potential for the hedonic skill game with singleton agents?

BRD can cycle when $q \geq 4$

3 agents with skill s_1, and 1 agent per skill s_i for all $i \in \{2, \ldots, 6\}$

7 tasks: $(78, \{s_1, s_2\})$, $(96, \{s_1, s_4\})$, $(114, \{s_2, s_3\})$, $(102, \{s_1, s_5\})$, $(54, \{s_1, s_6\})$, $(18, \{s_1, s_2, s_3\})$, $(180, \{s_1, s_3, s_4\})$

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A specific dynamics ($q = n$)

1. Compute a specific social optimum as the starting state
2. At each step, if at least one player has a profitable move, then use some specific rule to select the one who deviates

The specific dynamics is such that:

1. the starting state (social optimum) is built in polynomial time
2. the next deviator is chosen in linear time
3. at most n^2 deviations before convergence

The starting state (social optimum)

Reminder: each player has a single skill (singleton agent instances)

For all i, let $n(i)$ be the number of players of $\mathcal{N}$ having skill s_i

Suppose w.l.o.g. that $n(1) \geq n(2) \geq \ldots \geq n(k)$

The structure of a social optimum is as follows (can be built in polytime):

	C_1	C_2	C_3	$\ldots$	$C_{n(1)}$
s_1	1	1	1	$\ldots$	1
s_2	1	1	1	$\ldots$	0
s_3	1	1	0	$\ldots$	0
$\vdots$					
s_k	1	0	0	$\ldots$	0

Each task is performed its maximum number of times because every task t can be performed by at most $\min_{s_i \in S(t)} n(i)$ coalitions

Convergence to a Nash stable outcome

Ruled deviations (*details skipped*), in particular, favor **best** response where the arrival coalition has smallest index

Deviations are only from right to left in the table (no incentive to go right)

	C_1	C_2	C_3	C_4
s_1	1	1	1	1
s_2	1	1	1	0
s_3	1	0	0	0

Convergence to a Nash stable outcome

Ruled deviations (*details skipped*): for example, favor **best** response where the arrival coalition has smallest index

Deviations are only from right to left in the table (no incentive to go right)

	C_1	C_2	C_3	C_4
s_1	2	1	1	0
s_2	1	1	1	0
s_3	1	0	0	0

Convergence to a Nash stable outcome

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s_2	2	1	0	0
s_3	1	0	0	0

- The process must stop (convergence)
- Each agent makes at mot n moves ($O(n^2)$ steps in total)

Theorem: the hedonic skill game always admits a Nash stable outcome which can be built in polynomial time, when $q = n$

Singleton agents instances, $q = 2$

Start from the following social optimum

	C_1	C_2
s_1	$\lceil n(1)/2 \rceil$	$\lfloor n(1)/2 \rfloor$
s_2	$\lceil n(2)/2 \rceil$	$\lfloor n(2)/2 \rfloor$
$\vdots$	$\vdots$	$\vdots$
s_k	$\lceil n(k)/2 \rceil$	$\lfloor n(k)/2 \rfloor$

Only profitable deviations from C_2 to C_1 before convergence (at most $n/2$ moves)

Theorem: The hedonic skill game always admits a Nash stable outcome which can be built in polynomial time, when $q = 2$

Singleton tasks instances

One-to-one correspondence between skills and tasks: each skill s is associated with a task t_s of weight w_s requiring s

Agents can have more than one skill

Theorem: BRD always converges for every parameter q

Proof sketch: every instance is a *congestion game*, a large class of games always admitting an *exact potential function* (namely, Rosenthal's potential function)

Each (task,coalition) pair is associated with a resource

Players having skill s in the same coalition *compete* for the weight of t_s

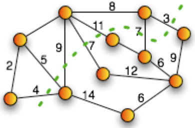
Singleton tasks instances

Corollary: A Nash stable outcome always exists and we know how to compute it (BRD)

Fast computation?

Theorem: Computing a Nash stable outcome in hedonic skill games with singleton tasks is a **PLS**-complete problem when $q = 2$

Proof sketch: **PLS**-reduction of MAX-CUT FLIP



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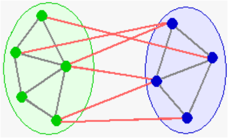

It's unlikely to find a local optimum (of the potential) in polynomial time, where local optimum=Nash stable outcome

Computing a social optimum

Theorem: Regarding the hedonic skill game with singleton tasks, maximizing the social welfare SW is an **NP**-hard problem when $q = 2$, and polynomial time solvable when $q = n$

Proof sketch:

Reduction of MAX-CUT for the case $q = 2$

1 coalition per player for the case $q = n$

Wrap up - 1

Hedonic skill game: coalitional game which admits a succinct representation

General instances of the hedonic skill game: no guarantee of a Nash stable outcome

Singleton agent instances:

- the existence of a Nash stable outcome is guaranteed and we can compute it polynomial time ($q \in \{2, n\}$)
- BRD may cycle ($q \geq 4$)
- a social optimum can be computed in polynomial time ($q \in \{2, n\}$)

Wrap up - 2

Singleton task instances

- the existence of a Nash stable outcome is guaranteed because BRD always converges ($\exists$ potential)
- computing a Nash stable outcome is a **PLS**-complete problem when $q = 2$, and a polynomial problem when $q = n$
- computing a social optimum is an **NP**-hard problem when $q = 2$, and a polynomial problem when $q = n$

Other results

In a general instance of the hedonic skill game, deciding if a Nash stable outcome exists is an **NP**-complete problem when $q \geq 3$ (reduction of the PARTITION problem)

We analyzed the Price of Anarchy of the hedonic skill game (almost tight results)

How bad (w.r.t. the social welfare) are the Nash stable outcomes, in the worst case?

Open problems - Future work

Singleton agent instances

- Convergence of BRD for $q \in \{2, 3\}$?
- Convergence of **Best** Response Dynamics (instead of **Better** Response Dynamics)? for which values of q?

Singleton tasks instances

- Polynomial time computation of an ε-**approximate** Nash stable outcome?

  Players deviate only if utility increases by a $(1 + \varepsilon)$ multiplicative factor

General instances

- Complexity of deciding if a Nash stable outcome exists when $q = 2$
- Other solution concepts (e.g. core stability)
- Other ways to share the weight of the tasks (burn money, non oblivious local search)

THANK YOU FOR YOUR ATTENTION