Multi-Criteria Decision Aiding in an industrial context

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GdR RO-IA: journée Industrielle

Outline

Decision

Decision Making:

- Multi-Criteria Decision Making (MCDM),
- Decision under uncertainty (DUU),
- Group Decision Making (GDM).

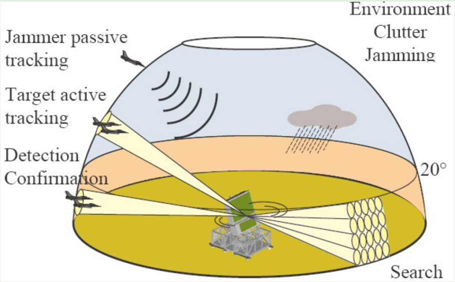
Example of applications at Thales

- Decision Aid for the Design of complex systems
- Decision function within an embedded system

Decision

Example of Decision function in embedded syst: radar management

- Too many tasks to be performed by a radar: search, tracking, . . .
- Use of multi-criteria decision function to allocate a priority to each task.



Decision

Needs for the 2 types of decisions

		Syst design	*Runtime*
Model	Model expressiveness • *Experts express complex and subtle decision strategies;* • *Need for expressive models*	++	+++
Post-decision	Explanation of the decision • *Very important for decision maker to understand the recommendation*	+++	+

Outline

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Notation

MCDA setting

- $N = \{1, \dots, n\}$: index set of attributes, descriptors.
- X_i: set of values representing attribute i (for $i \in N$).
- $X = X_1 \times \cdots \times X_n$: set of alternatives/acts.
- $\succsim$: preference relation of the DM over X

Notation

- $(x_A, y_{-A}) \in X$: compound alternative $z \in X$ such that $z_i = x_i$ if $i \in A$ and y_i otherwise

The leading model in MCDA: the additive utility

Additive Utility model

$$x \succsim y \iff U(x) \geq U(y) \quad , \; U(x) = \sum_{i \in N} u_i(x_i)$$

The Additive Utility model is characterized by the Independence property.

Independence

$A \subseteq N$ is said to be **independent** for $\succsim$ if for all $x_A, y_A \in X_A$ and all $z_{-A}, t_{-A} \in X_{-A}$

$$(x_A, z_{-A}) \succsim (y_A, z_{-A}) \quad \iff \quad (x_A, t_{-A}) \succsim (y_A, t_{-A}).$$

Interpretation: common values on other attributes do not affect preference on A.

The leading model in MCDA: the additive utility

Example (Choice of a car)

3 criteria:

- Colour,
- Type of the car,
- Price

The attributes are preferentially independent if

$$(\textbf{Red}, \textbf{Sport Car}, 50\,k\,Euros) \succ (\textbf{Gray}, \textbf{Vintage car}, 50\,k\,Euros)$$

$$\iff (\textbf{Red}, \textbf{Sport Car}, 20\,k\,Euros) \succ (\textbf{Gray}, \textbf{Vintage car}, 20\,k\,Euros)$$

(this relation shall be true for every subset of criteria, and every common values on the other attributes)

Hence **Red Sport Car** is intrinsically preferred to **Gray Vintage car** ceterus paribus (all else being equal)

Outline

Violation of independence

Example (Investment selection)

Option	Investment profile	Age	Innovation
A	**Niche**	**Old**	**Low**
B	**Niche**	**New**	**High**
C	**Competitive**	**Old**	**Low**
D	**Competitive**	**New**	**High**

Decision strategies:

- $A \succ B$: for *niche* market, the *age* is the most important criterion;
- $C \prec D$: for *competitive* market, *innovation* is clearly a leading criterion.

$\Rightarrow$ **Violation of independence assumption**

Example of interaction among criteria

Engineering of complex system

Two types of criteria:

- *F*: functional criteria to measure Measures of Performance, Measures of Success. Use of simulation, system models,. . .
- *NF*: non-functional criteria, such as OpEx, CapEx, TRL,. . .

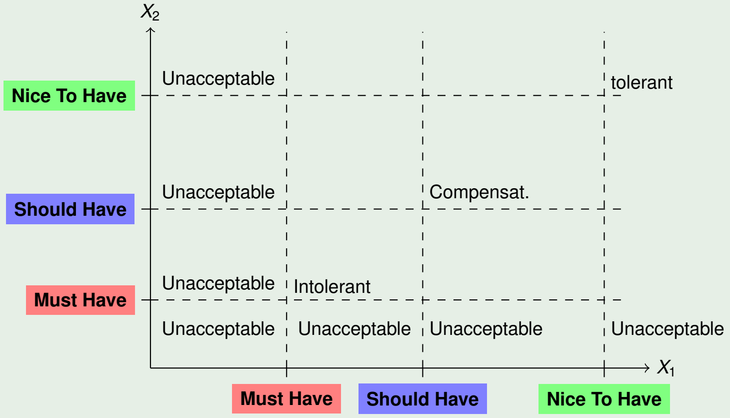
Decision maker's preferences:

- If the functional criteria are not met, then the customer will find the solution unacceptable
  - A very good performance on the non-functional criteria cannot compensate a bad assessment on the functional criteria
- When functional criteria are met, the customer looks carefully at price.

This cannot be represented by an additive model.

Violation of independence

Example (Engineering problem)



Outline

1. Context & Motivations
2. Decision Model
   - Additive Utility
   - Violation of independence
   - Weak independence
   - GAI model
3. Explanation
4. Synthesis

Towards the definition of partial preference over X_i

Weak independence

$\succsim$ is said to be **weakly independent** if $\{i\}$ is independent for $\succsim$, for all $i \in N$.
Hence, for all $i \in N$, all $x_i, y_i \in X_i$ and all $z_{-i}, t_{-i} \in X_{-i}$

$$(x_i, z_{-i}) \succsim (y_i, z_{-i}) \quad \Longleftrightarrow \quad (x_i, t_{-i}) \succsim (y_i, t_{-i}).$$

Partial preference $\succsim_i$

If $\succsim$ is weakly independent, then we define $\succsim_i$ by: for all $x_i, y_i \in X_i$

$$x_i \succsim_i y_i \quad \text{if} \quad (x_i, z_{-i}) \succsim (y_i, z_{-i}) \; \forall z_{-i} \in X_{-i}.$$

Remark: then x_i is at least as good as y_i ceteris paribus.

Towards the definition of partial preference over X_i

Violation of weak independence: car example

3 criteria:

- Colour,
- Type of the car,
- Price

Preferences of the expert:

$$(\textbf{Red}, \textit{Sport Car}, 50\,k\ \textit{Euros}) \succ (\textbf{Gray}, \textit{Sport Car}, 50\,k\ \textit{Euros})$$

$$\iff (\textbf{Red}, \textit{Family car}, 20\,k\ \textit{Euros}) \prec (\textbf{Gray}, \textit{Family car}, 20\,k\ \textit{Euros})$$

Hence **Red** color is not intrinsically preferred to **Gray** color ceterus paribus (all else being equal)!
Hence weak independence is not satisfied for attribute X_1.

Towards the definition of partial preference over X_i

Proposition

Let $\succsim$ be a weakly independent weak order on X. Then:

- $\succsim_i$ exists and is a weak order,
- $[x_i \succsim_i y_i \ \forall i \in N] \quad \Rightarrow \quad x \succsim y$

Outline

Generalized Additive Independence (GAI) model

car example

Preferences of the expert:

$$(\textbf{Red}, \textit{Sport Car}, 50\,k\,\textit{Euros}) \succ (\textbf{Gray}, \textit{Sport Car}, 50\,k\,\textit{Euros})$$

$$\Longleftrightarrow (\textbf{Red}, \textit{Family car}, 20\,k\,\textit{Euros}) \prec (\textbf{Gray}, \textit{Family car}, 20\,k\,\textit{Euros})$$

- This cannot be represented by an additive utility: $u_1(\text{Red}) > u_i(\text{Gray})$ by the 1st relation and $u_1(\text{Red}) < u_i(\text{Gray})$ by the 2nd relation.

Generalized Additive Independence (GAI) model

Car Example revisited

- What would do a specialist in Additive Utility to solve this problem?
  Group some attributes together: Attributes 1 and 2 interact together.
- New model:

$$x \succsim y \iff u_{1,2}(x_1, x_2) + u_3(x_3) \geq u_{1,2}(y_1, y_2) + u_3(y_3)$$

- With values

$$\begin{array}{ll} u_{1,2}(\text{Red}, \text{Sport Car}) = 6 & u_{1,2}(\text{Gray}, \text{Sport Car}) = 5 \\ u_{1,2}(\text{Red}, \text{Family car}) = 3 & u_{1,2}(\text{Gray}, \text{Family car}) = 4 \\ u_3(20\,k\;Euros) = 2 & u_3(50\,k\;Euros) = 1 \end{array}$$

Then the previous preferences are met.

Generalized Additive Independence (GAI) model

Generalized Additive Independence (GAI) model [*Fishburn'67, Bacchus et al'95*]

The Generalized Additive Independence (GAI) model takes the form

$$U(x) = \sum_{S \in \mathcal{S}} u_S(x_S)$$

where

- $\mathcal{S}$ is a set of subsets of N
- $u_S : X_s \to \mathbb{R}$

Example

$$\text{Car example: } \mathcal{S} = \Big\{\{1,2\},\{3\}\Big\}$$

$$\text{Overlapping case: } \mathcal{S} = \Big\{\{1,2\},\{2,3\}\Big\}$$

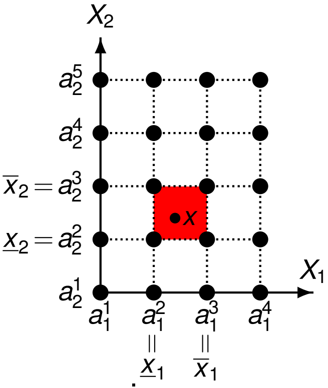
Unknowns of the GAI model

Definition

- Discretization of X_i:
  $\widehat{X}_i = \{a_i^1, a_i^2, \ldots, a_i^{p_i}\} \subseteq X_i$
- $\widehat{X}_S = \prod_{i\in S} \widehat{X}_i$
- $v := \{v_S(z_S) \ : \ S \in \mathcal{S} \ , \ z_S \in \widehat{X}_S\}$

Definition (Interpolation)

$$u_S(x_S) = \sum_{A\subseteq S} \left[\prod_{i\in A} \frac{\overline{x}_i - x_i}{\overline{x}_i - \underline{x}_i} \times \prod_{i\in S\setminus A} \frac{x_i - \underline{x}_i}{\overline{x}_i - \underline{x}_i} \times v_S(\underline{x}_A, \overline{x}_{S\setminus A}) \right],$$



We write $u_S(x_S) = \sum_{\widehat{x}_S \in \widehat{X}_S} coef_{S,x_S}(\widehat{x}_S)\ v_S(\widehat{x}_S)$.

Learning setting

Definition (preferential information)

- $x^q \succeq y^q$, where $x^q, y^q \in X$ and $q \in Q$, means that x^q is judged at least as good as y^q

Learning datum $x^q \succeq y^q$ is transformed into the linear constraint

$$T_q(v) := \sum_{S \in \mathcal{S}} \sum_{\widehat{x}_S \in \widehat{X}_S} \left(coef_{S, x_S^q}(\widehat{x}_S) - coef_{S, y_S^q}(\widehat{x}_S) \right) v_S(\widehat{x}_S) \geq 0$$

Definition (Learning setting)

Find v such that

$$U \text{ is monotone}$$
$$T_q(v) \geq 0 \qquad \forall q \in Q.$$

Monotonicity conditions

Definition (Monotonicity conditions)

$\forall x \in X \; \forall i \in N \; \forall y_i \in X_i$ with $y_i \succsim_i x_i$

$$U(y_i, x_{N \setminus \{i\}}) \geq U(x)$$

The number of elementary conditions is

$$\sum_{i \in N} \left[(p_i - 1) \times \prod_{j \in N \setminus \{i\}} p_j \right].$$

Example

- $n = 10$, 5 unknowns per attribute ($p_i = 5$)
- $\mathcal{S}$: all singletons and pairs of attributes.
- Overall number of unknowns is 1125
- Number of monotony constraints: 78 125 000.

Conditions on u_S

Definition (First condition)

In MCDM, we assume that $U(x) \geq 0$ for all $x \in X$

In [Greco, Mousseau, Slowinski'2012], the terms u_S (with $|S| = 2$) can take both positive and negative signs.

Example

- Consider $U(x_1, x_2) = 2\, x_1 + x_2 - \max(x_1, x_2)$, which has a negative term.
- From the relation, $\min(x_1, x_2) + \max(x_1, x_2) = x_1 + x_2$,
- Equivalent expression: $U(x_1, x_2) = x_1 + \min(x_1, x_2)$.

Conditions on u_S

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Assertion

- Is it possible that $u_S(x_S) \geq 0$ for each $S \in \mathcal{S}$?
- Is it possible that each u_S is monotone?

2-additive GAI model

Definition (2-additive GAI model)

The GAI model is said to be 2-additive if $\mathcal{S}$ is composed of only singletons and pairs.

Theorem [*Labreuche, Grabisch. MSS, 2017*]

Let us consider a 2-additive GAI model U that is monotone and non-negative.
Then there exists non-negative and monotone functions $u_{i,j} : X_i \times X_j \to [0,1]$ (for every $\{i,j\} \subseteq N$) and non-negative and monotone functions $u_i : X_i \to [0,1]$ (for every $i \in N$) such that for all $x \in X$

$$U(x) = \sum_{i=1}^{n} U_i(x_i) + \sum_{i,j} U_{i,j}(x_i, x_j).$$

2-additive GAI model

Consequence of the Theorem

Instead of enforcing monotonicity condition on U, it is sufficient to impose monotonicity conditions on each term u_S.
For every $S \in \mathcal{S}$ we assume that

(i) $\forall x_S \in \widehat{X}_S$, $u_S(x_S) \geq 0$,

(ii) u_S is monotonic w.r.t. $\succsim_i$ for all $i \in S$.

Assumption on u_S

Example

Consider $p_i = 4$ and a 2-additive GAI model. The following chart give the number of monotonicity constraints.

n	4	6	8	10	12
# of monotonicity constraints with U	2 000	75 000	2 500 000	78 125 000	2 343 750 000
# of monotonicity constraints with Th.	256	624	1 152	1 840	2 688

n	14	16	18
# of monotonicity constraints with U	68 359 375 000	$1.95313E + 12$	$5.49316E + 13$
# of monotonicity constraints with Th.	3 696	4 864	6 192

Outline

Why shall we explain decisions?

Needs

- EU "General Data Protection Regulation" (2016):
  - ask the "right to explanation" for algorithmic decision-making
  - Due date: May 2018
- Interpretation of the model:
  - increase acceptance of user
- Explanation of a particular decision:
  - Defend the decision to stakeholders

Explanation of the Additive Utility Model

Existing works

Reference	Description
[*Klein. 1994*] [*Carenini, Moore. Artificial Intelligence, 2006*]	Compute the degree of influence of each criterion in the decision; and select the most ppropriate ones according to this index
[*Labreuche. Artificial Intelligence, 2011*]	Identify predefined situations and create dedicated explanation schemas in each situation
[*Belacene,Labreuche, Maudet, Mousseau, Ouerdane. T&D, 2017*]	Transform the comparison by a equivalent sequence of simplest comparisons

Definition of an explanation

Definition [*Belacene,Labreuche, Maudet, Mousseau, Ouerdane. T&D, 2017*]

An explanation of $z \succsim z'$ is a sequence

$$z = z[1] \succsim z[2] \succsim \cdots \succsim z[q] = z'$$

s.t. for each k: $z[k]$ and $z[k+1]$ are equal on all attributes except 2.

Remark

This is close to even swap [*Hammond, Keeney, Raiffa'98*].

Additive Utility model

car example

$$(Red, Sport\ Car, 50\,k\ Euros) \succ (Gray, Family\ car, 25\,k\ Euros)$$

because

(i) $(Red, Sport\ Car, 50\,k\ Euros) \succ (Gray, Sport\ Car, 45\,k\ Euros)$ as

$$(\textbf{Red}, \star, \textbf{50 k Euros}) \succ (\textbf{Gray}, \star, \textbf{45 k Euros})$$

(ii) $(Gray, Sport\ Car, 45\,k\ Euros) \succ (Gray, Family\ car, 25\,k\ Euros)$ as

$$(\star, \textbf{Sport Car}, \textbf{45 k Euros)} \succ (\star, \textbf{Family car}, \textbf{25 k Euros})$$

Outline

Synthesis

Critical points for MCDA

- Why using MCDA tools?:
  - Why not simply using Excell?
  - Wy not using spider radars?

MCDA added-value

- Methodology:
  - Frame the problem
  - Find the finite set of relevant attributes
  - Take the heat out of the debate among several decision makers
  - Set “rules of the game”; make the assessment method explcit and transparent
  - Why not using spider radars?