Multi-stage stochastic programming approach for a lot-sizing problem with remanufacturing

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joint work with

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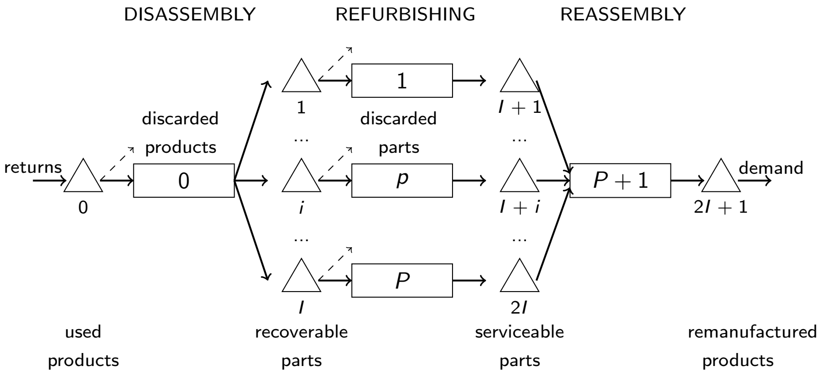
A three-echelon remanufacturing system



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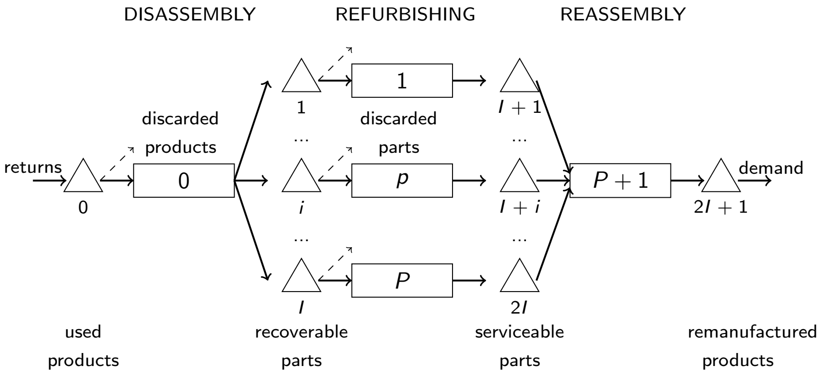
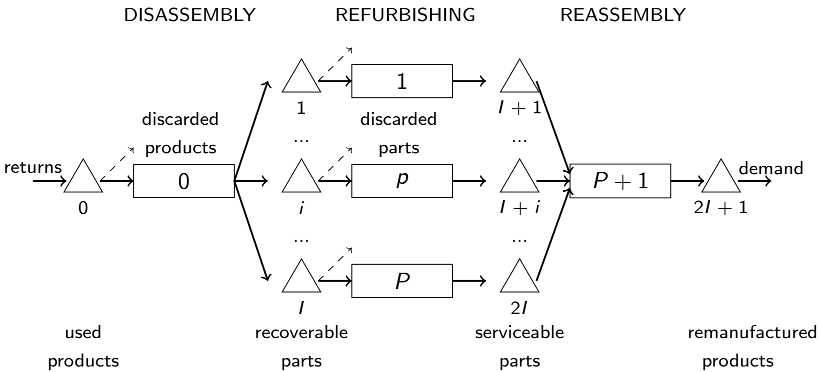


Industrial applications: mobile phones, electrical equipment...

Jayaraman [2006], Franke et al. [2006], Han et al. [2013], Ahn et al. [2011]

Assumptions



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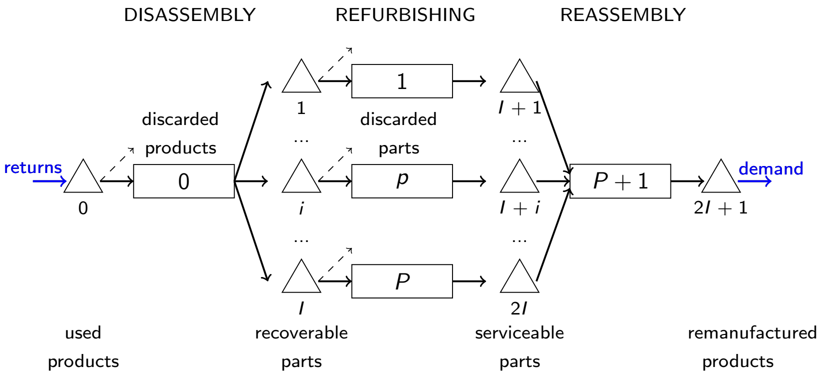
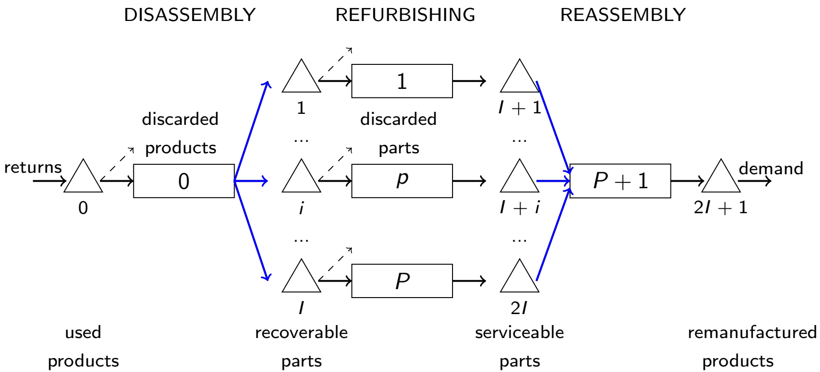


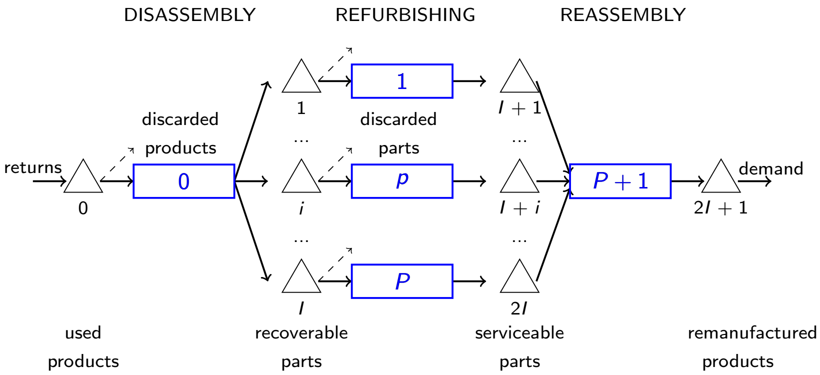
1. a single type of product

Assumptions



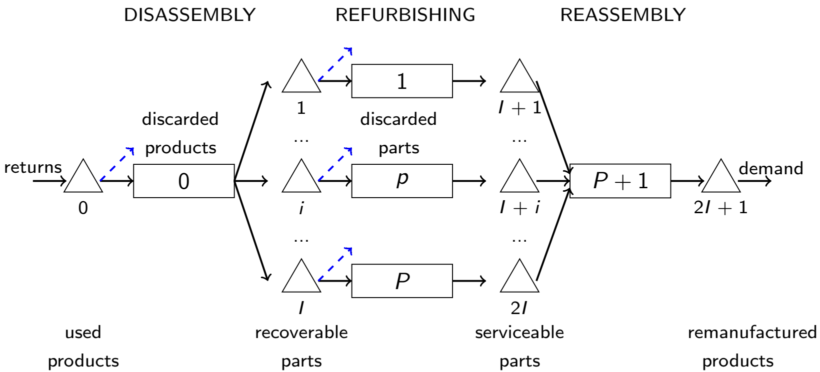
1. a single type of product
2. identical bill-of-materials

Assumptions



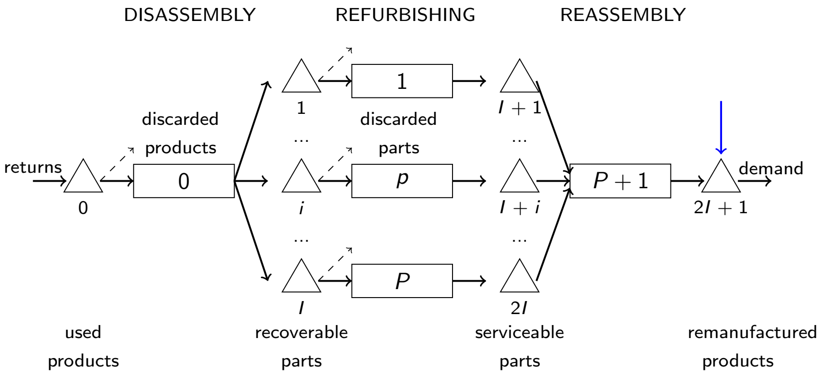
1. a single type of product
2. identical bill-of-materials
3. uncapacitated production processes

Assumptions



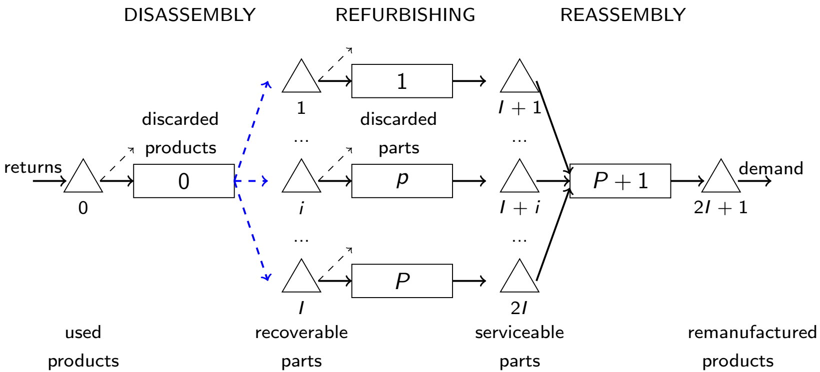
1. a single type of product
2. identical bill-of-materials
3. uncapacitated production processes
4. discard items

Assumptions



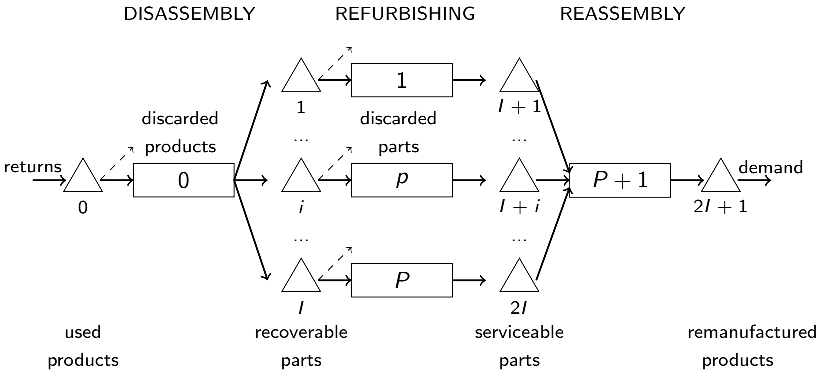
1. a single type of product
2. identical bill-of-materials
3. uncapacitated production processes
4. discard items
5. unmet demand is lost

Assumptions



1. a single type of product
2. identical bill-of-materials
3. uncapacitated production processes
4. discard items
5. unmet demand is lost
6. yield of the disassembly process

Sources of uncertainty



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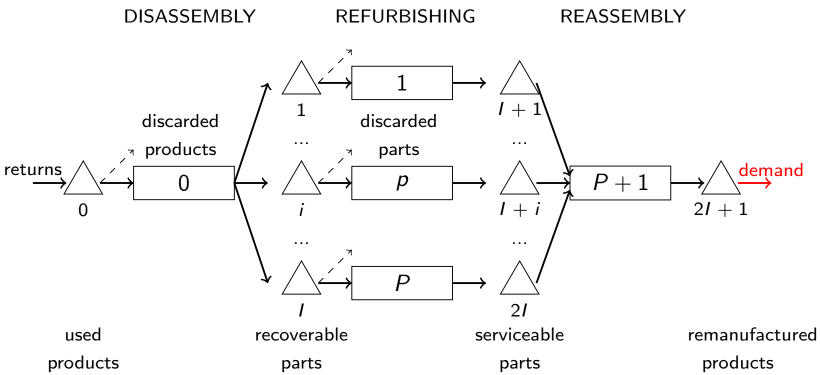

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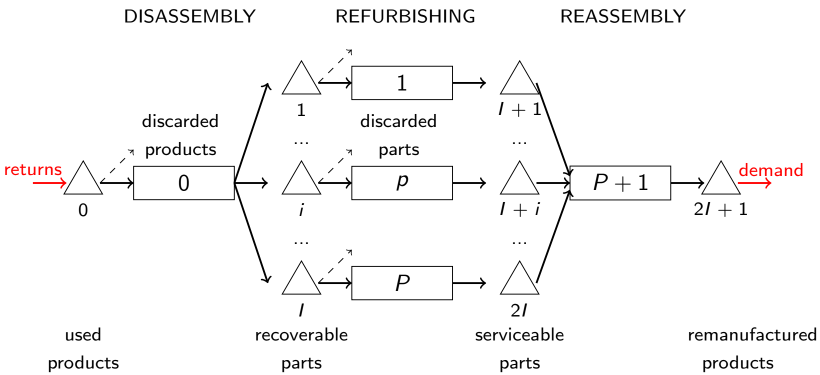

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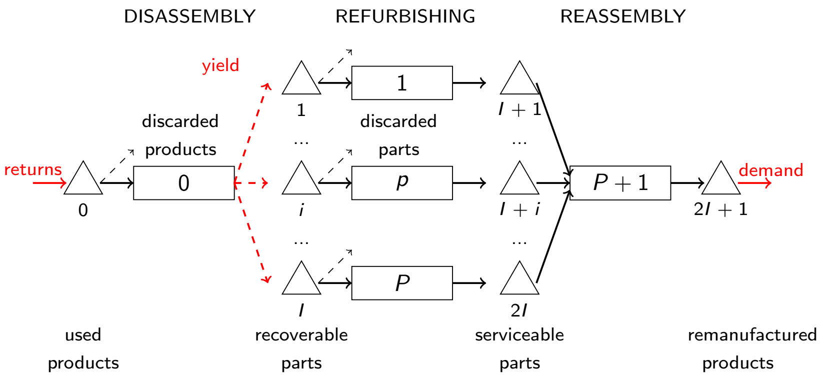

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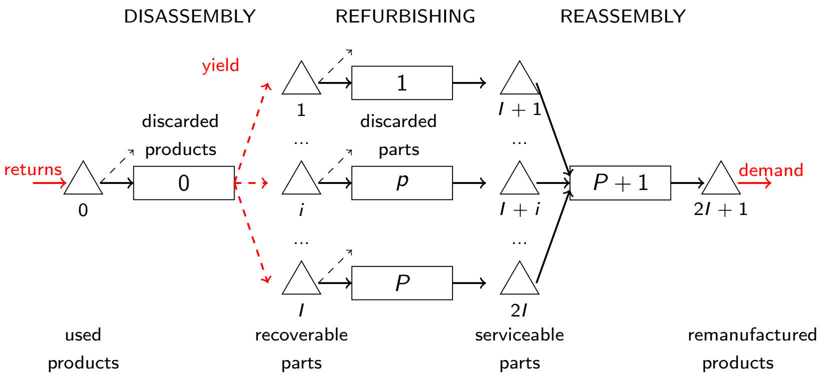

Ilgin and Gupta [2010], Lage Junior and Filho [2012], Lage Junior and Filho [2016]

Multi-stage stochastic programming

Assumption

- Not all decisions have to be made before uncertainty realization.
- Some can be postponed to a future point in time.

Dynamic multi-stage decision process

Multi-stage stochastic programming

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Dynamic multi-stage decision process

Observation of random parameters for stage 1

Multi-stage stochastic programming

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Dynamic multi-stage decision process

Observation of random parameters for stage 1 $\longrightarrow$ Production planning decisions for stage 1

Multi-stage stochastic programming

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Dynamic multi-stage decision process

Observation of random parameters for stage 1 $\longrightarrow$ Production planning decisions for stage 1 $\longrightarrow$ Observation of random parameters for stage 2

Multi-stage stochastic programming

Assumption

- Not all decisions have to be made before uncertainty realization.
- Some can be postponed to a future point in time.

Dynamic multi-stage decision process

Observation of random parameters for stage 1 $\longrightarrow$ Production planning decisions for stage 1 $\longrightarrow$ Observation of random parameters for stage 2 $\longrightarrow$ Production planning decisions for stage 2

Multi-stage stochastic programming

Assumption

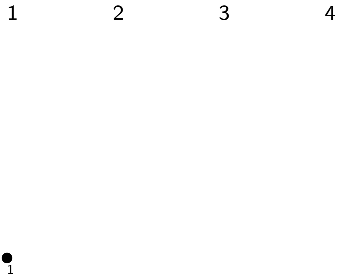
- Not all decisions have to be made before uncertainty realization.
- Some can be postponed to a future point in time.

Dynamic multi-stage decision process

Observation of random parameters for stage 1 $\rightarrow$ Production planning decisions for stage 1 $\rightarrow$ Observation of random parameters for stage 2 $\rightarrow$ Production planning decisions for stage 2 $\rightarrow$ $\cdots$ $\rightarrow$ Observation of random parameters for stage S $\rightarrow$ Productions planning decisions for stage S

Scenario tree

Scenario tree



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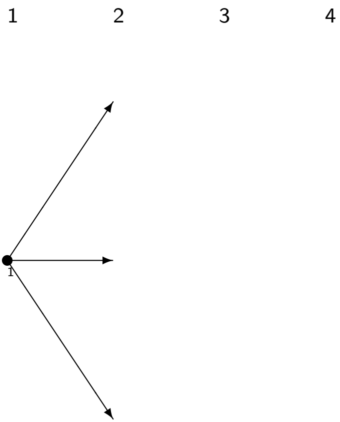

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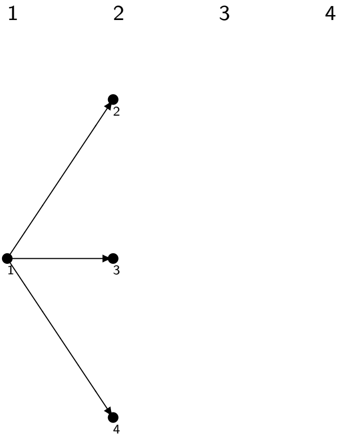

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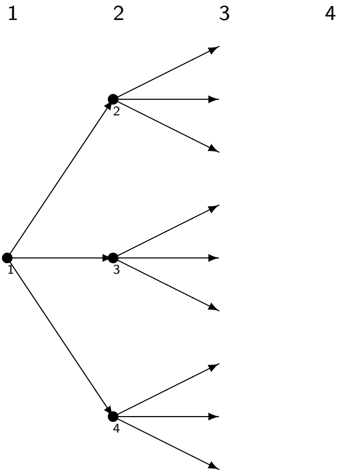

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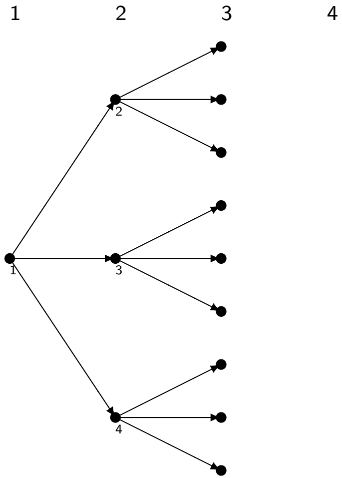

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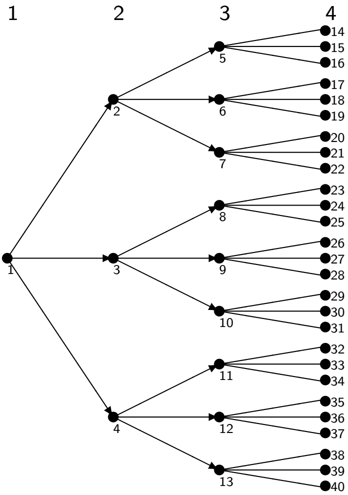

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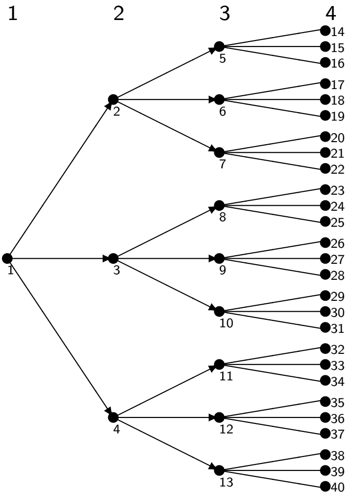
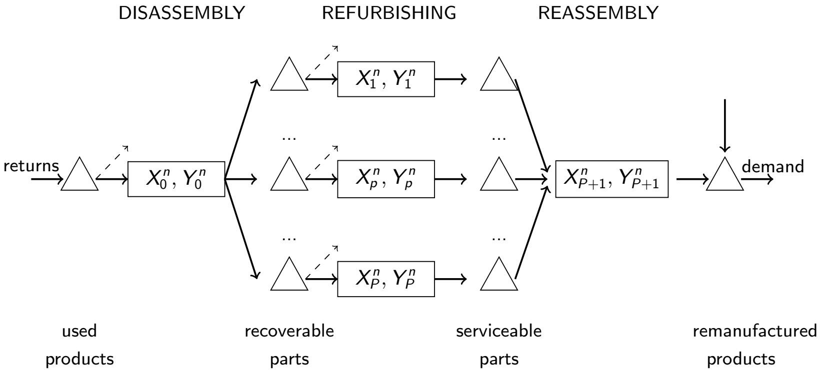


Scenario tree is defined by a set of $\mathcal{V}$ nodes.

Decision variables

for every node $n \in \mathcal{V}$:



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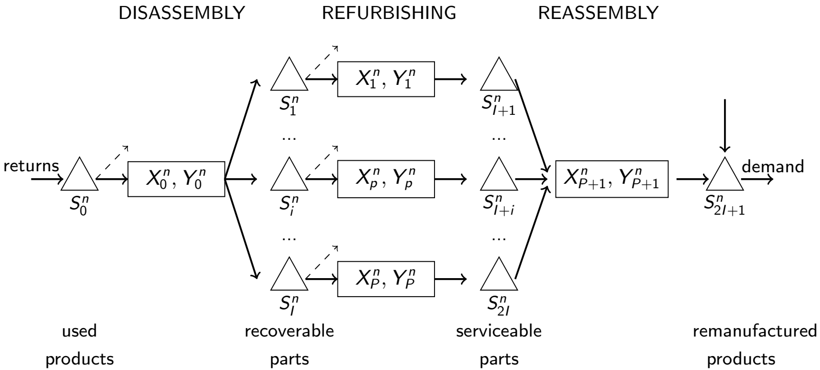

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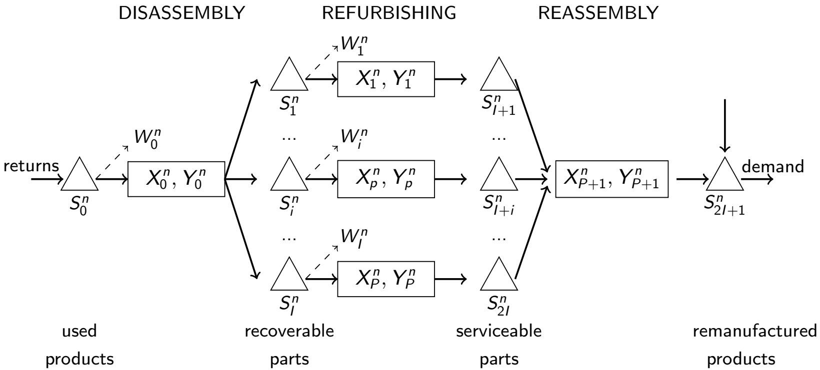

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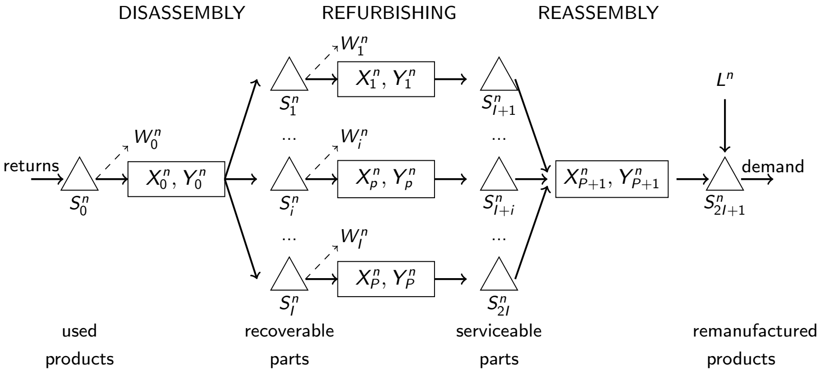

MILP formulation

$$\min \sum_{n\in\mathcal{V}} \Big(\sum_{p\in\mathcal{J}} f_p^n Y_p^n + \sum_{i\in\mathcal{I}} h_i^n S_i^n + l^n L^n + \sum_{i\in\mathcal{I}_r\cup\{0\}} q_i^n W_i^n + g^n X_0^n \Big)$$

setup/inventory/discarding/lost sales costs

subject to:

MILP formulation

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setup/inventory/discarding/lost sales costs

subject to:

$$X_p^n \leq M_p^n Y_p^n \qquad \forall p \in \mathcal{J}, \forall n \in \mathcal{V}$$

Setup-production

MILP formulation

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Setup-production

$$S_0^n = S_0^{n-1} + r^n - X_0^n - W_0^n \qquad \forall n \in \mathcal{V}$$

Inventory balance

MILP formulation

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$$S_{2I+1}^n = S_{2I+1}^{n-1} + X_{I+1}^n - d^n + L^n \qquad \forall n \in \mathcal{V}$$

Inventory balance

MILP formulation

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Inventory balance

$$S_i^0 = 0 \qquad \forall i\in\mathcal{I}$$

$$S_i^n, W_i^n, L^n \ge 0 \qquad \forall i\in\mathcal{I}, \forall n\in\mathcal{V}$$

$$X_p^n \ge 0,\ Y_p^n \in \{0,1\} \qquad \forall p\in\mathcal{J}, \forall n\in\mathcal{V}$$

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$$\min \sum_{n\in\mathcal{V}} \Big(\sum_{p\in\mathcal{J}} f_p^n Y_p^n + \sum_{i\in\mathcal{I}} h_i^n S_i^n + l^n L^n + \sum_{i\in\mathcal{I}_r\cup\{0\}} q_i^n W_i^n + g^n X_0^n \Big)$$

subject to:

$$
\begin{aligned}
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& X_p^n \geq 0,\ Y_p^n \in \{0,1\} && \forall p \in \mathcal{J}, \forall n \in \mathcal{V}
\end{aligned}
$$

Difficulties:

MILP formulation

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\end{aligned}$$

Difficulties:

Dependent demand
→ Dependence between echelons

MILP formulation

$$\min \sum_{n\in\mathcal{V}} \Big(\sum_{p\in\mathcal{J}} f_p^n Y_p^n + \sum_{i\in\mathcal{I}} h_i^n S_i^n + l^n L^n + \sum_{i\in\mathcal{I}_r\cup\{0\}} q_i^n W_i^n + g^n X_0^n \Big)$$

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\end{aligned}$$

Difficulties:

Setup constraints
$\rightarrow$ Poor linear relaxation

Dependent demand
$\rightarrow$ Dependence between echelons

MILP formulation

$$\min \sum_{n\in\mathcal{V}} \Big(\sum_{p\in\mathcal{J}} f_p^n Y_p^n + \sum_{i\in\mathcal{I}} h_i^n S_i^n + l^n L^n + \sum_{i\in\mathcal{I}_r\cup\{0\}} q_i^n W_i^n + g^n X_0^n \Big)$$

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Difficulties:

Setup constraints
→ Poor linear relaxation

Dependent demand
→ Dependence between echelons

Solutions:

Echelon stock = Total inventory of an item in the system, as such or as a component of other items.

Pochet and Wolsey [2006]

New valid inequalities

Echelon stock reformulation

$$\min \sum_{n\in\mathcal{V}} \Big(\sum_{p\in\mathcal{J}} f_p^n Y_p^n + \sum_{i\in\mathcal{I}} h_i^n S_i^n + l^n L^n + \sum_{i\in\mathcal{I}_r\cup\{0\}} q_i^n W_i^n + g^n X_0^n \Big)$$

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& E_i^n = E_i^{n-1} + \pi_i^n \alpha_i X_0^n - d^n + L^n - W_i^n && \forall i \in \mathcal{I}_r, \forall n \in \mathcal{V} \\
& E_i^n = E_i^{n-1} + X_{i-I}^n - d^n + L^n && \forall i \in \mathcal{I}_s, \forall n \in \mathcal{V} \\
& E_{2I+1}^n = E_{2I+1}^{n-1} + X_{I+1}^n - d^n + L^n && \forall n \in \mathcal{V}
\end{aligned}$$

Echelon stock reformulation

$$\min \sum_{n\in\mathcal{V}} \Big(\sum_{p\in\mathcal{J}} f_p^n Y_p^n + \sum_{i\in\mathcal{I}} h_i^n S_i^n + l^n L^n + \sum_{i\in\mathcal{I}_r\cup\{0\}} q_i^n W_i^n + g^n X_0^n \Big)$$

subject to:

$$
\begin{array}{lr}
X_p^n \le M_p^n Y_p^n & \forall p \in \mathcal{J}, \forall n \in \mathcal{V} \\
S_0^n = S_0^{n-1} + r^n - X_0^n - W_0^n & \forall n \in \mathcal{V} \\
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E_i^n = E_i^{n-1} + X_{i-l}^n - d^n + L^n & \forall i \in \mathcal{I}_s, \forall n \in \mathcal{V} \\
E_{2l+1}^n = E_{2l+1}^{n-1} + X_{l+1}^n - d^n + L^n & \forall n \in \mathcal{V} \\
E_i^n - E_{i+l}^n \ge 0 & \forall i \in \mathcal{I} \\
E_{i+l}^n - E_{2l+1}^n \ge 0 & \forall i \in \mathcal{I} \\
E_i^n \ge 0 & \forall i \in \mathcal{I}, \forall n \in \mathcal{V} \\
E_i^0 = 0 & \forall i \in \mathcal{I} \\
W_i^n, L^n \ge 0 & \forall i \in \mathcal{I}, \forall n \in \mathcal{V} \\
X_p^n \ge 0,\ Y_p^n \in \{0,1\} & \forall p \in \mathcal{J}, \forall n \in \mathcal{V}
\end{array}
$$

Linking constraints

Echelon stock reformulation

$$\min \sum_{n\in\mathcal{V}} \Big(\sum_{p\in\mathcal{J}} f_p^n Y_p^n + \sum_{i\in\mathcal{I}} h_i^n S_i^n + l^n L^n + \sum_{i\in\mathcal{I}_r\cup\{0\}} q_i^n W_i^n + g^n X_0^n \Big)$$

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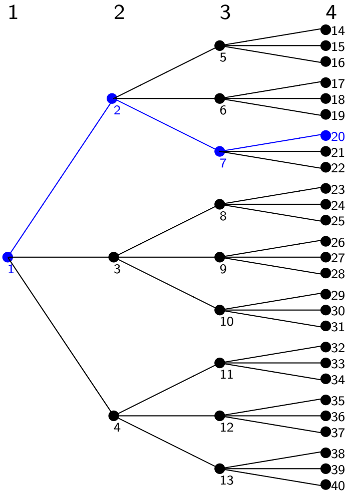
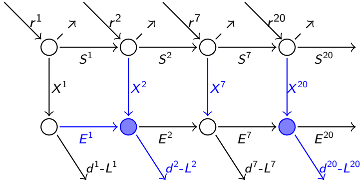
$P+1$ independent single-item single-echelon lot-sizing problems with lost sales

} Linking constraints

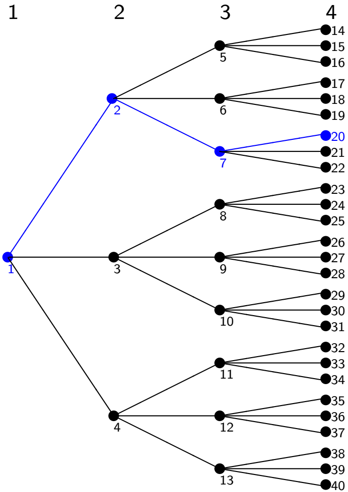
Path inequalities

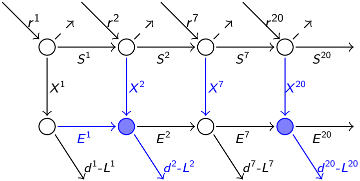




Given $k = 1$ and a subset $U = \{2, 20\}$.

Path inequalities





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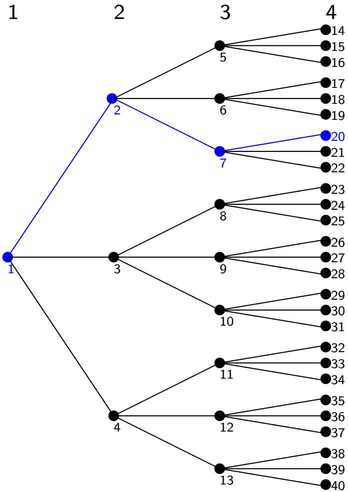
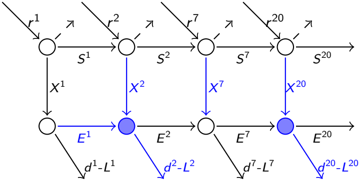
$$E^1 + X^2 + X^7 + X^{20} \geq (d^2 - L^2) + (d^{20} - L^{20})$$

Path inequalities





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$$E^1 + X^2 + X^7 + X^{20} \geq (d^2 - L^2) + (d^{20} - L^{20})$$

$$E^k + Y^2(d^2 + d^{20}) + Y^7(d^{20}) + Y^{20}(d^{20}) \geq \sum_{t \in U} (d^n - L^n)$$

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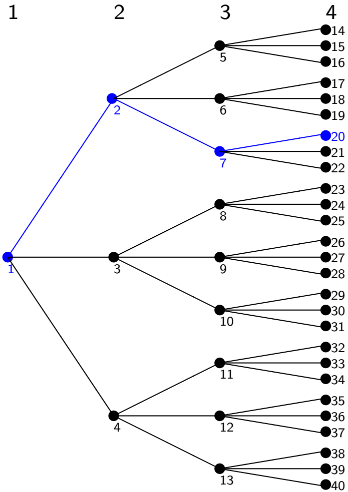

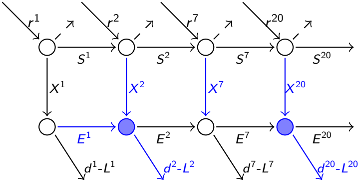

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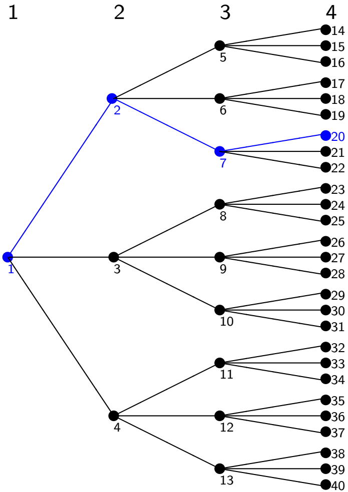

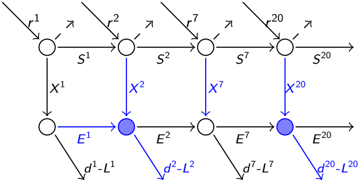

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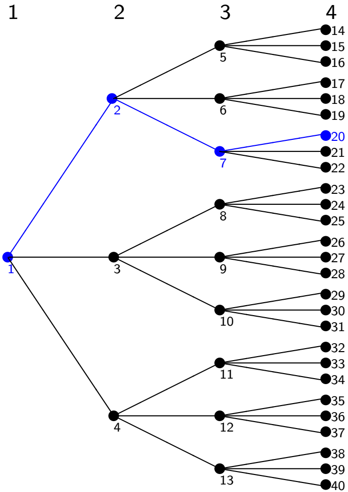
Loparic et al. [2001]

Path inequalities



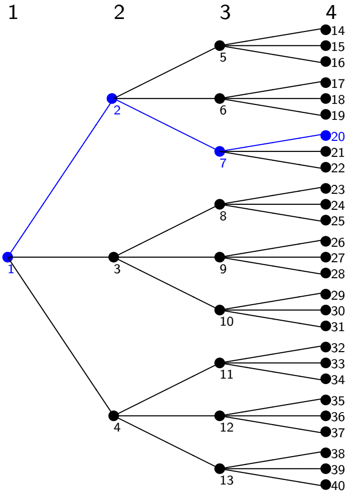
Let k be a non-leaf node and l be a leaf node. Let $U \in \mathcal{P}(k,l)$.

$$E^k_{p+l} \geq \sum_{n \in U} \left[d^n \left(1 - \sum_{\nu \in \mathcal{P}(k,n)} Y^{\nu}_p - L^n\right)\right]$$

where:

- $\mathcal{P}(n,m)$: path between node n and m in the scenario tree

Path inequalities



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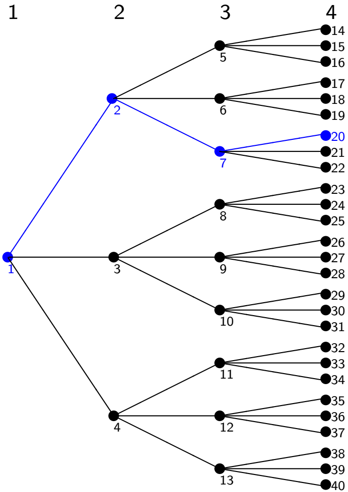
Separation
Polynomial in $\mathcal{O}(|\mathcal{V}|^2)$

Path inequalities



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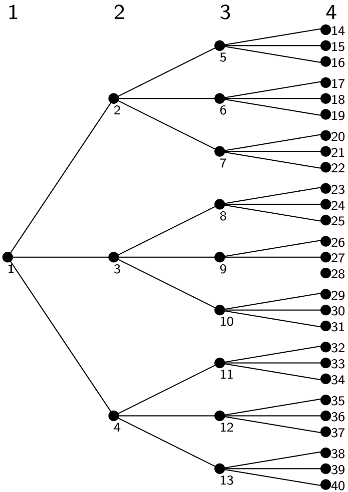
Separation
Polynomial in $\mathcal{O}(|\mathcal{V}|^2)$

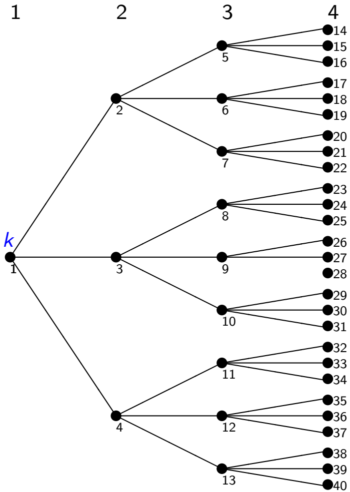
Key point in practice
Careful selection of the valid inequalities to be added to the formulation

Cutting-planes generation algorithm



Two main ingredients:

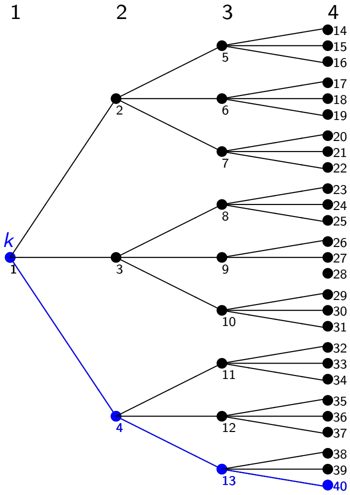
Cutting-planes generation algorithm



Two main ingredients:

For each $k \in \mathcal{V}$, we search for:

Cutting-planes generation algorithm



Two main ingredients:

For each $k \in \mathcal{V}$, we search for:

$\rightarrow$ the most violated valid inequality.

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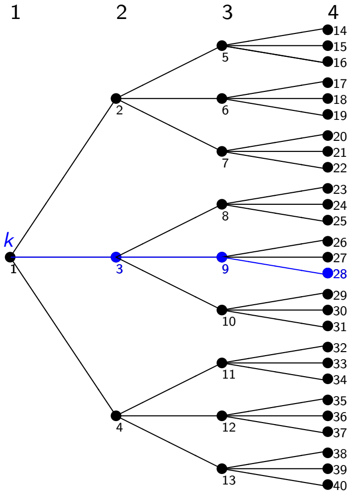

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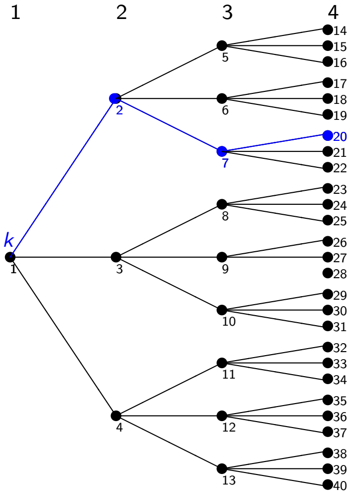

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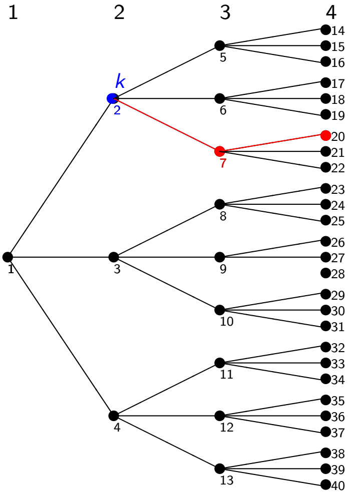
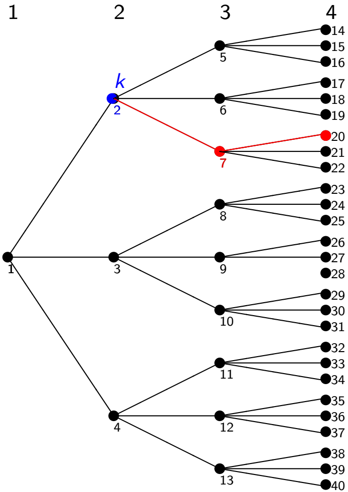


Two main ingredients:

For each $k \in \mathcal{V}$, we search for:

- $\rightarrow$ the most violated valid inequality.
- $\rightarrow$ avoiding the addition of inequalities involving similar subsets of set-up variables.

Cutting-planes generation algorithm



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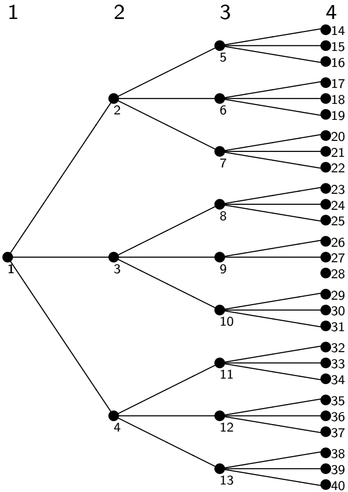

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- → all valid inequalities are still potentially considered for inclusion in the formulation.

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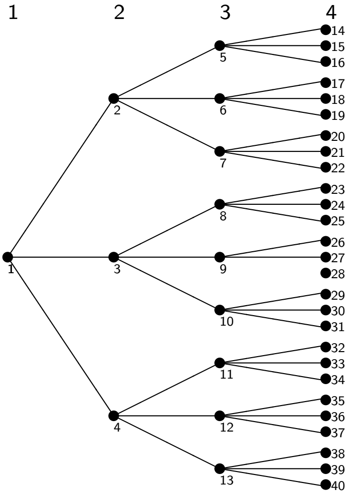

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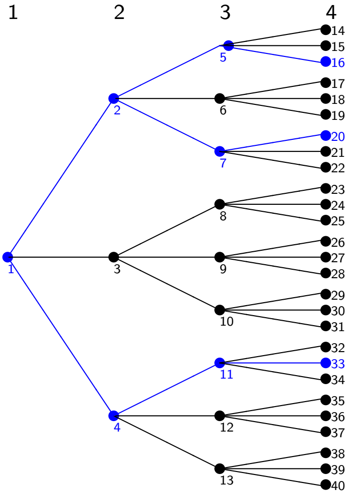
- → the separation problem is solved exactly.

From Path to Tree inequalities



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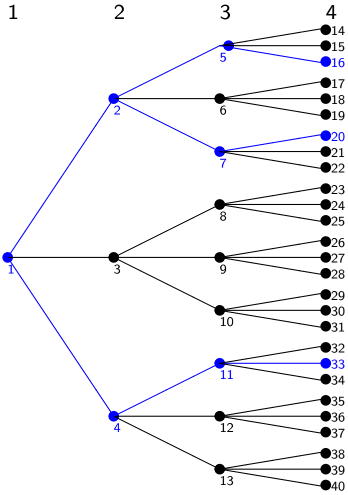

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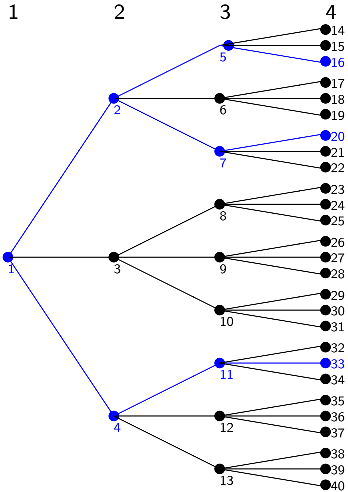

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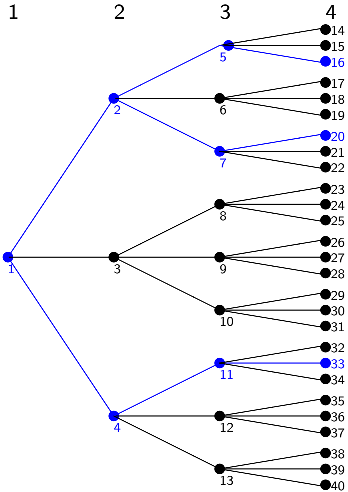

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Separation
Exact separation: very long computation times

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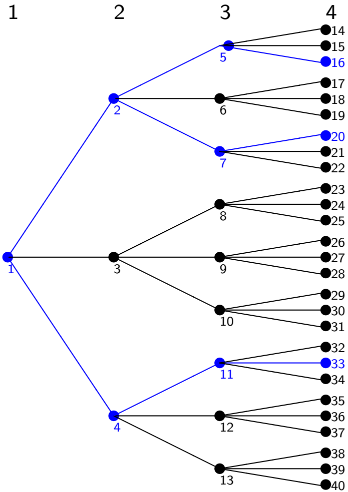

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Separation
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$\rightarrow$ Heuristic separation algorithm based on a neighborhood search

Instances Generation and Branch-and-cut algorithms

Random generation based on the numerical values used by [Ahn *et al*, 2011] and [Jayaraman, 2006]

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Branch-and-Cut algorithms:

- CPLEX: the generic branch-and-cut algorithm embedded in CPLEX 12.8.
- Path and Tree: a customized branch-and-cut algorithm using the (k, U) Path inequalities and the newly introduced (k, U) Tree inequalities to strengthen the echelon-stock formulation.

Numerical results

Instances		CPLEX default				Path and Tree			
I	Nodes	Gap_{LP}	Gap_{MIP}	Time	# Opt	Gap_{LP}	Gap_{MIP}	Time	# Opt
5	381								
	1022								
10	381								
	1022								
	1365								

- Total of 3600 randomly generated instances
- Average values for 720 randomly generated instances
- Resolution with CPLEX 12.6.1 on a PC running under Windows 10, Intel Core i7, 8 GB of RAM
- Time limit : 900s

- Results published in *Computers & Operations Research*. Quezada et al. [2020].

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	1022	11.04	0.45	900.29	0				
10	381	7.92	0.30	897.49	9				
	1022	10.17	3.11	901.38	0				
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10	381	7.92	0.30	897.49	9	1.20	0.19	817.73	103
	1022	10.17	3.11	901.38	0	1.38	0.84	900.91	0
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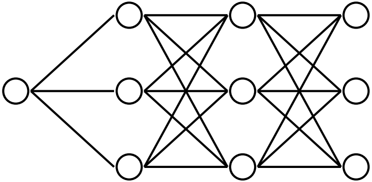

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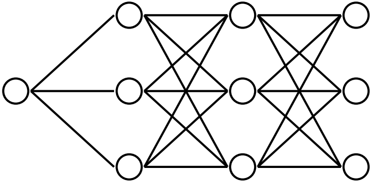

- Polyhedral recourse function in LP setting.
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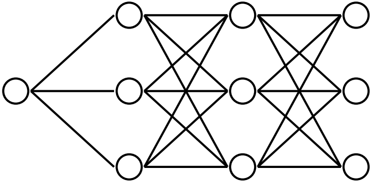

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However, we are trying to solve an MILP.

Zou et al. [2019] introduced the SDDiP algorithm.

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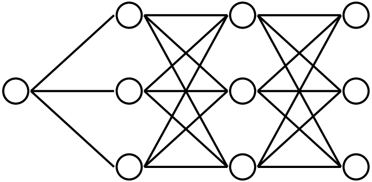

- Polyhedral recourse function in LP setting.
$\rightarrow$ Benders' cuts.

However, we are trying to solve an MILP.

Zou et al. [2019] introduced the SDDiP algorithm.

- Key Assumption: Binary state variables

SDDiP algorithm

- First introduced by Pereira and Pinto [1991]

- Key assumption: stage-wise independent process

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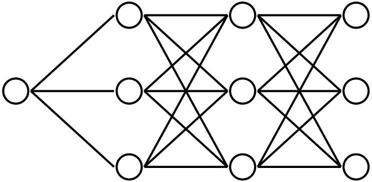

- Polyhedral recourse function in LP setting.
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However, we are trying to solve an MILP.

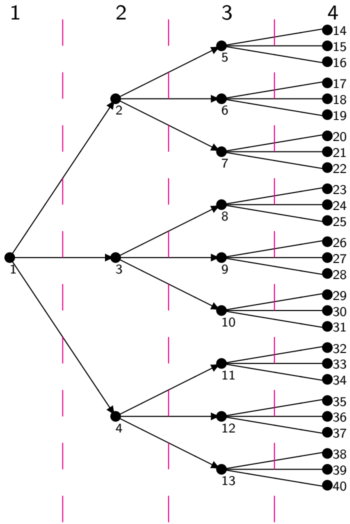
Zou et al. [2019] introduced the SDDiP algorithm.

- Key Assumption: Binary state variables

-**Our contribution**:

→ We propose a new extension of the SDDiP algorithm for solving multistage stochastic lot-sizing problems.

Dynamic programming formulation: a full decomposition



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Decomposes the original problem into a series of single-node sub-problems

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Decomposes the original problem into a series of single-node sub-problems

Reformulation to use SDDiP

For each node $n \in \mathcal{V}$:

$$Q^n(u^{a^n}) := \min F^n(X^n, Y^n, S^n, W^n, L^n) + \mathcal{Q}^t(u^n)$$

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Binary approximation

$$u_i^{n,\lambda} \in \{0,1\} \qquad \forall i \in \mathcal{I}, \forall \lambda \in \mathcal{B}$$

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Copy Constraint

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Binary approximation

Copy Constraint

Subproblems involve a larger number of binary variables

Approximate subproblems

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& \sigma_i^n \geq 0 && \forall i \in \mathcal{I} \\
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\end{aligned}
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Approximation of $\mathcal{Q}^t(\cdot)$ by Strengthened Benders' Cuts

Strengthened Benders' Cuts

For each node $n \in \mathcal{V}$:

$$\hat{R}^n(S^{a^n}) := \min_{y,x,s,\sigma} F^n(X^n, Y^n, S^n, W^n, L^n) + \sum_{i \in \mathcal{I}} \pi_i^n (S_i^{a^n} - \sigma_i^n) + \psi^t(S^n)$$

subject to:

$$(X^n, Y^n, S^n, X^n, L^n) \in \mathcal{X}^n$$

$$\sigma_i^n \geq 0 \qquad \forall i \in \mathcal{I}$$

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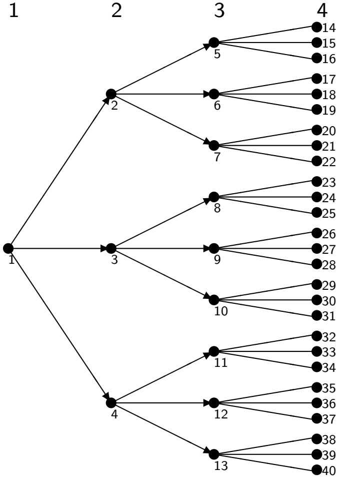
Dynamic programming formulation: a partial decomposition



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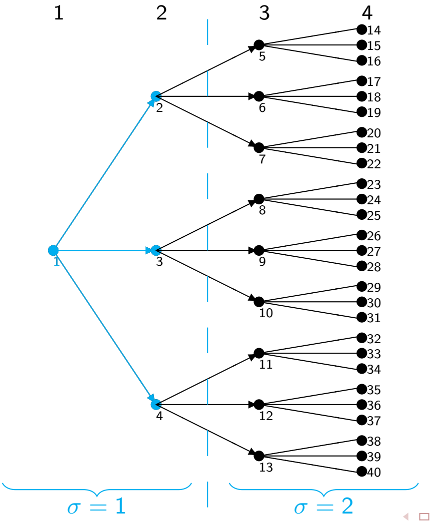

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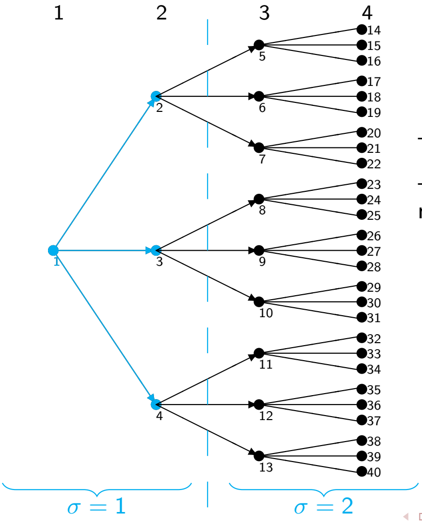

-Set of root nodes $\mho$

-Subtree defined by a set of nodes $\mathcal{W}^\eta$

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- Reduced number of expected cost-to-go functions to approximate.
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Additional Strengthened Benders' Cuts

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$$\sigma_p^\eta \geq 0 \qquad \forall p \in \mathcal{I}$$

$$S_p^k \geq \sum_{n \in U} [d^n(1 - \sum_{\nu \in \mathcal{P}(k,n)} Y_p^\nu - L^n)] \qquad \forall U \in \mathcal{P}(k,l), k, l \in \mathcal{W}^\eta$$

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$$\hat{R}^n(S^{a^n}) := \min \sum_{n \in \mathcal{W}^\eta} \rho^n F^n(X^n, Y^n, S^n, W^n, L^n) + \sum_{p \in \mathcal{I}} \pi_p^n (S_p^{a^n} - \sigma_p^n) + \sum_{\ell \in \mathfrak{L}(\eta)} \psi^t(S^\ell)$$

subject to:

$$(X^n, Y^n, S^n, W^n, L^n) \in \mathcal{X}^n \qquad \forall n \in \mathcal{W}^\eta$$

$$\sigma_p^\eta \geq 0 \qquad \forall p \in \mathcal{I}$$

$$S_p^k \geq \sum_{n \in U} [d^n(1 - \sum_{\nu \in \mathcal{P}(k,n)} Y_p^\nu - L^n)] \qquad \forall U \in \mathcal{P}(k,l), k, l \in \mathcal{W}^\eta$$

$$\psi^\ell(S^\ell) := \min\{\theta^\ell : \theta^\ell \geq \sum_{m \in \mathcal{C}(n)} \rho^{nm}(v_l^m + (\pi_l^m)^\intercal \sigma^\ell)\} \qquad \forall l = 1, ..., i$$

$$X^n, S^n, W^n, L^n \geq 0; \; Y^n \in \{0, 1\} \qquad \forall p \in \mathcal{I}, n \in \mathcal{W}^\eta$$

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Leads to a better approximation of the expected cost-to-go functions in practice.

Illustrative example

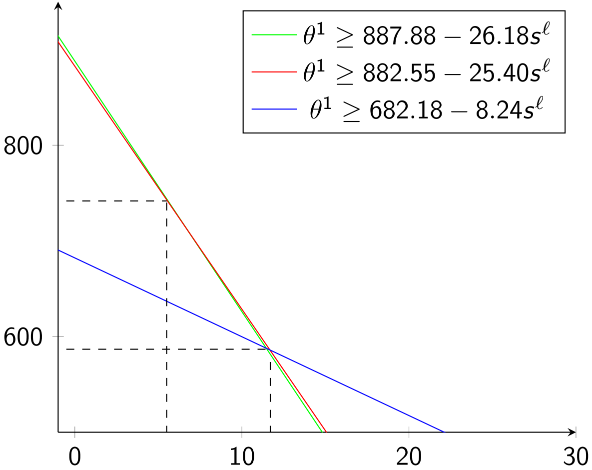

Figure: Illustration of different SB cuts.

Illustration of extSDDiP

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At each iteration i:

- Sampling step

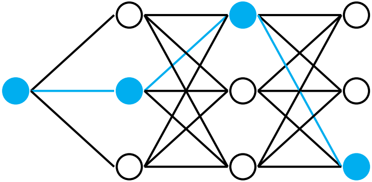

Illustration of extSDDiP

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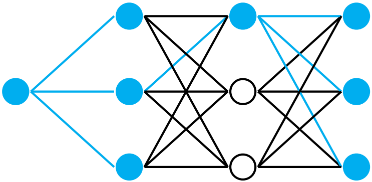
- Sampling step
- Forward step

$$\underline{Q}_i^{\eta}(S_i^{a^{\eta}}) := \min \sum_{n \in \mathcal{W}^{\eta}} \rho^n F^n(X^n, Y^n, S^n, W^n, L^n) + \sum_{\ell \in \mathcal{L}(\eta)} \psi_i^t(S^{\ell})$$



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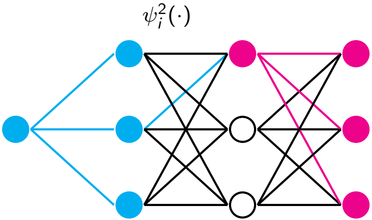

- Backward step

$\rightarrow$ Cutting-plane generation phase.

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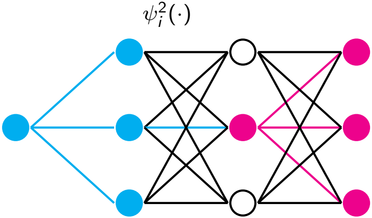

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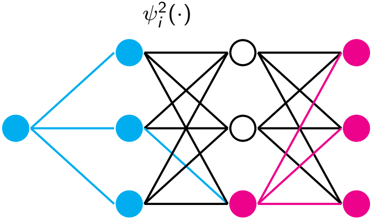

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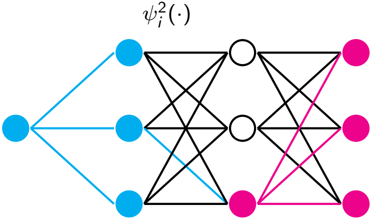

Output: Lower and Upper bound

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Random generation based on the numerical values used by Guan et al. [2009]

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Numerical results

Instance				CPLEX	SDDiP	AppSDDiP	ExtSDDiP
Σ	R	b	#*Scen*	Gap (Time)	Gap (Time)	Gap (Time)	Gap (Time)
4	10	1	1,000				
	20	1	8,000				
6	10	1	100,000				
	20	1	3,200,000				
8	5	2	78,125				
	5	5	78,125				
12	3	1	177,000				
	3	3	177,000				
Average							

- Total of 480 randomly generated instances
- Average values for 60 randomly generated instances
- Resolution with CPLEX 12.8
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Σ	R	b	#*Scen*	Gap (Time)	Gap (Time)	Gap (Time)	Gap (Time)
4	10	1	1,000	0.24 (6,513)			
	20	1	8,000	1.57 (7,201)			
6	10	1	100,000	68.27 (7,217)			
	20	1	3,200,000	- (-)			
8	5	2	78,125	93,48 (7,236)			
	5	5	78,125	- (-)			
12	3	1	177,000	91,33 (7,243)			
	3	3	177,000	- (-)			
Average				50.98 (5,638)			

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4	10	1	1,000	0.24 (6,513)	23.55 (4,306)		
	20	1	8,000	1.57 (7,201)	22.88 (4,202)		
6	10	1	100,000	68.27 (7,217)	30.37 (5,641)		
	20	1	3,200,000	- (-)	35.67 (5,608)		
8	5	2	78,125	93,48 (7,236)	47.76 (7,151)		
	5	5	78,125	- (-)	41.74 (7,222)		
12	3	1	177,000	91,33 (7,243)	51.04 (7,207)		
	3	3	177,000	- (-)	55.60 (7,230)		
Average				50.98 (5,638)	38.57 (6,071)		

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Σ	R	b	#*Scen*	Gap (Time)	Gap (Time)	Gap (Time)	Gap (Time)
4	10	1	1,000	0.24 (6,513)	23.55 (4,306)	7.46 (1,358)	
	20	1	8,000	1.57 (7,201)	22.88 (4,202)	6.98 (1,974)	
6	10	1	100,000	68.27 (7,217)	30.37 (5,641)	8.81 (2,513)	
	20	1	3,200,000	- (-)	35.67 (5,608)	9.83 (3,664)	
8	5	2	78,125	93,48 (7,236)	47.76 (7,151)	9.11 (3,929)	
	5	5	78,125	- (-)	41.74 (7,222)	6.68 (3,815)	
12	3	1	177,000	91,33 (7,243)	51.04 (7,207)	11.00 (3,313)	
	3	3	177,000	- (-)	55.60 (7,230)	9.72 (3,607)	
Average				50.98 (5,638)	38.57 (6,071)	8.70 (3021)	

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4	10	1	1,000	0.24 (6,513)	23.55 (4,306)	7.46 (1,358)	1.18 (1,956)
	20	1	8,000	1.57 (7,201)	22.88 (4,202)	6.98 (1,974)	4.70 (3,463)
6	10	1	100,000	68.27 (7,217)	30.37 (5,641)	8.81 (2,513)	5.61 (4,579)
	20	1	3,200,000	- (-)	35.67 (5,608)	9.83 (3,664)	7.59 (4,814)
8	5	2	78,125	93,48 (7,236)	47.76 (7,151)	9.11 (3,929)	5.64 (4,579)
	5	5	78,125	- (-)	41.74 (7,222)	6.68 (3,815)	4.11 (5,884)
12	3	1	177,000	91,33 (7,243)	51.04 (7,207)	11.00 (3,313)	6.29 (3,595)
	3	3	177,000	- (-)	55.60 (7,230)	9.72 (3,607)	5.65 (5.987)
Average				50.98 (5,638)	38.57 (6,071)	8.70 (3021)	5.10 (4357)

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  → Stage-wise independence assumption.

Acknowledgment

We would like to thank the Programa de Cooperación Científica ECOS-ANID PC23E10-ECOS230013 for their support and funding, which made this research and presentation possible.


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