The pioneering spirit



Designing a robust supply chain of relief material for disaster preparedness with aftershocks: a branch-and-cut approach

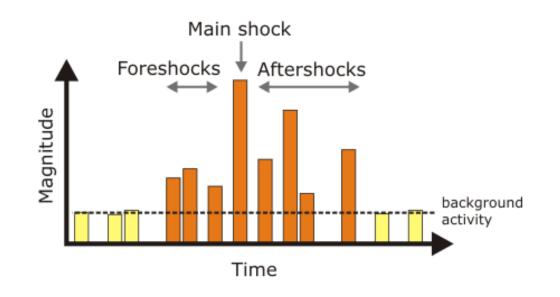
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POC 13/12/2024, CNAM Paris

Earthquakes and Aftershocks

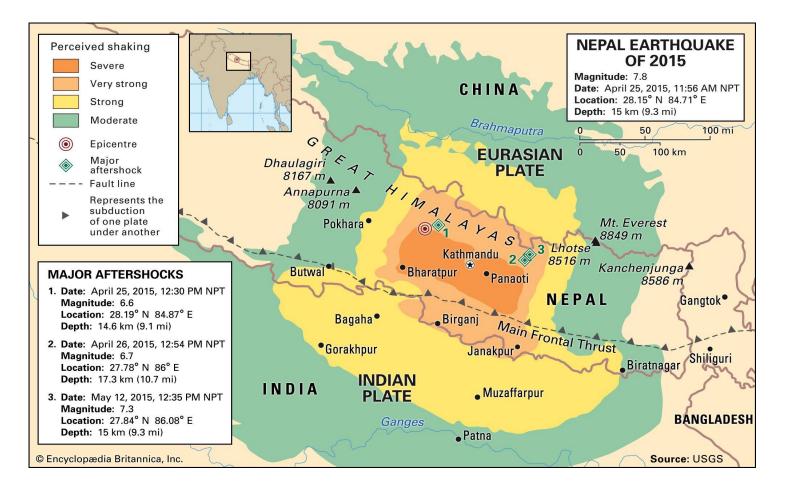
- Aftershocks are earthquakes that follow the largest shock of an earthquake sequence
- They are typically smaller than the main shock, but the larger the main shock the larger and more numerous the aftershocks
- They can continue over a period of weeks, months, or even years after the main shock
- They can also trigger other emergencies like landslides, building collapses, tsunamis, etc.

 This paper: focus on earthquakes, but can be generalized to any kind of disaster with aftershocks





Motivation - Nepal Earthquake (2015)



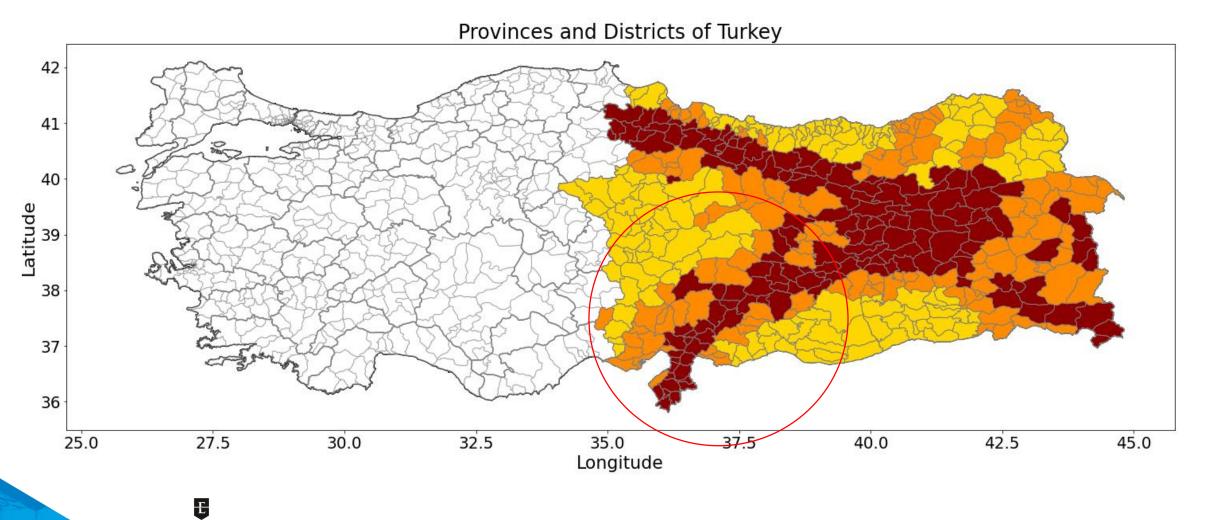
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- Caused landslides, avalanches, building collapses
- 3 major aftershocks followed
- Deaths of approximately 9000
 people, 16,800 injured, 2.8 million
 people displaced due to the main
 earthquake and the aftershocks
- The *last aftershock* alone resulted in over 200 deaths and over 2500 injured
- An *avalanche* caused by the earthquake killed 19 on Mt. Everest and stranded hundreds at the base camp

Case Study on Turkey-Syria (Feb. 2023)

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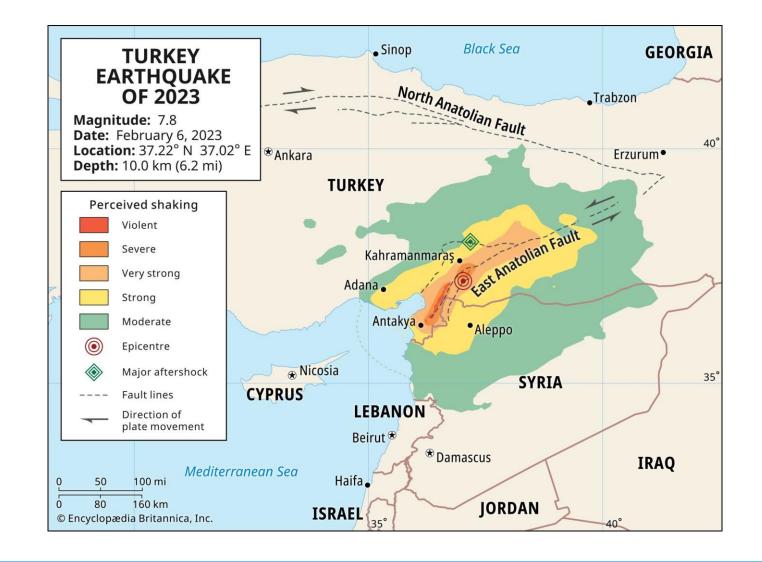


Case Study on Turkey-Syria (Feb. 2023)

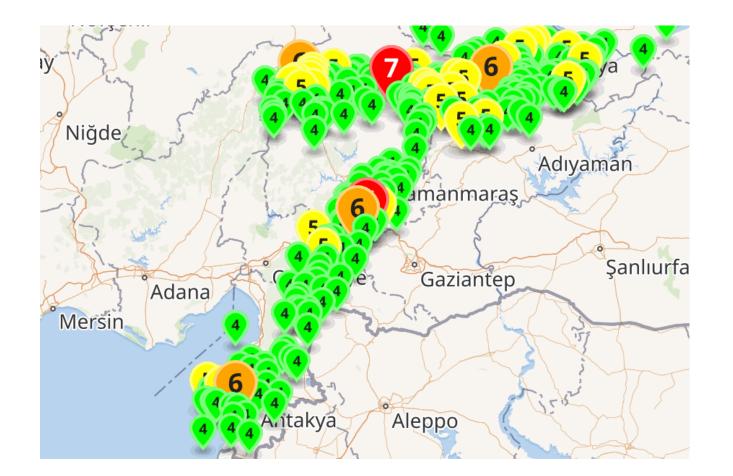
- Turkey and Syria were hit by a series of earthquakes
- First magnitude 7.8 earthquake on February 6

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Case Study on Turkey-Syria (Feb. 2023)



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- + 570 aftershocks recorded within 24h of the M_w 7.8 earthquake and over 30,000 recorded by May 2023.
- +25 aftershocks M_w >= 4 recorded within 6 h of the main earthquake
- Some regions were also hit by floods in the following months
- Syria already undergoing a humanitarian crisis

Facility Location decisions- Problem setting

- Disaster Preparedness Phase- Location of Facilities
- Strategic level of decision-making (Supply Chain network design)
- Prepositioning of relief materials, e.g., blankets, water, canned food, first-aid
- Earthquakes (1st-stage scenarios) with Aftershocks (2nd stage) in its 'vicinity'
- To serve the demands of an area (Demand nodes) in a catastrophic event
- *Robust Facility Location* Worst-case situation in the event of an earthquake with at most Δ aftershocks
- Uncapacitated Facilities (Warehouses)

Assumptions:

- No damage to Facilities due to earthquakes : located in "safe" places (or even in risky areas for earthquakeproof buildings)
- 2nd stage demand is caused by the closest aftershock
- Demand d at a "demand" node = f(population, distance to the earthquake or aftershock, magnitude) : d1,d2



Literature Review

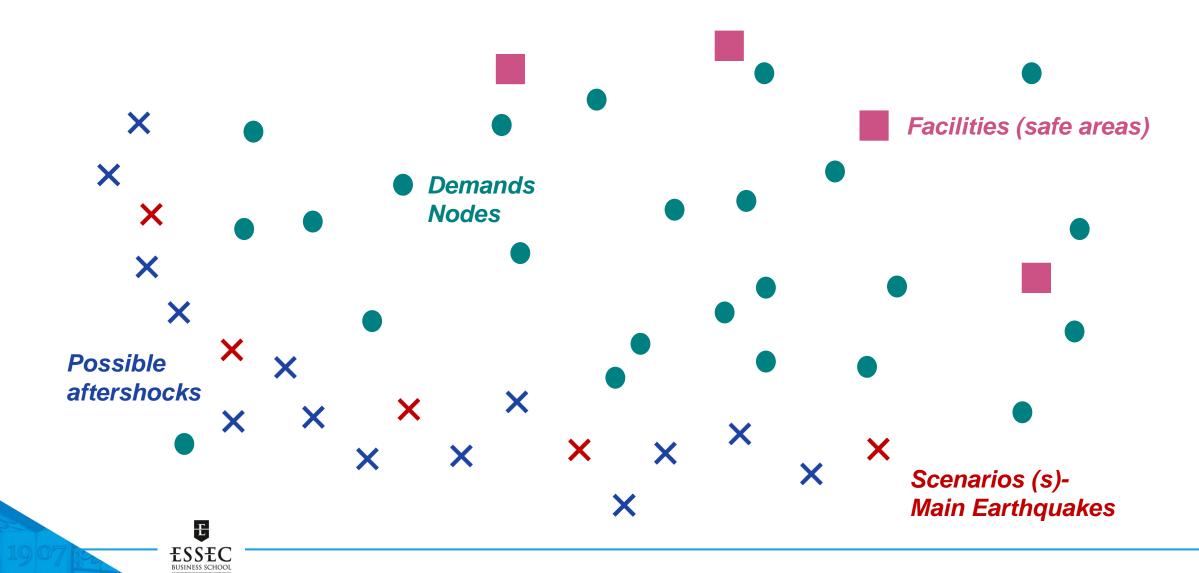
- Facility location with Prepositioning- Balcik and Beamon (2008); Elci and Noyan (2018); Rezaei-Malek et al. (2016); Balcik et al. (2019)
- Facility location-allocation decisions- Paul and MacDonald (2016); Elci and Noyan (2018); Paul and Wang (2019)
- Facility location with disruptions- Salman and Yucel (2015); Paul and MacDonald (2016)
- Other types of facilities: shelter sites, relief distribution centers, casualty collection points, etc.- Lin et al. (2012); Lu and Sheu (2013); Alizadeh et al. (2019)
- Simultaneous disasters- Ozbay (2018); Ozbay et al. (2019)

Facility location under uncertainty: Mainly three kinds- uncertainty on the supply side, uncertainty on the demand side, uncertainties in the network (Donmez et al. 2021)

We focus on uncertainty in the location of the disasters (specifically aftershocks), which results in demand uncertainty

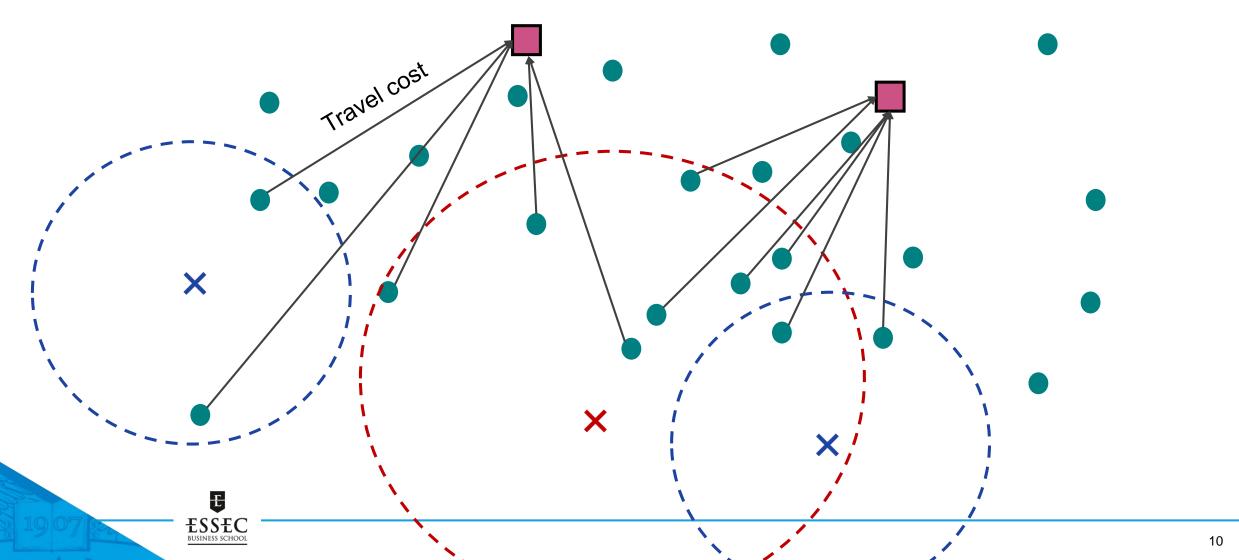


Facility Location decisions- Problem setting



Facility Location decisions- Problem setting

(travel cost = travel time x demand; demand depends on closest shock location) Whatever the demand, damaged cities are served by the closest open warehouse (uncapacitated)



Uncertainty set for aftershocks

$$\mathscr{K}_{s}^{\Delta} = \{ K \subset \mathscr{K}_{s} : |K| \leq \Delta \}, \ s \in \mathscr{S}$$

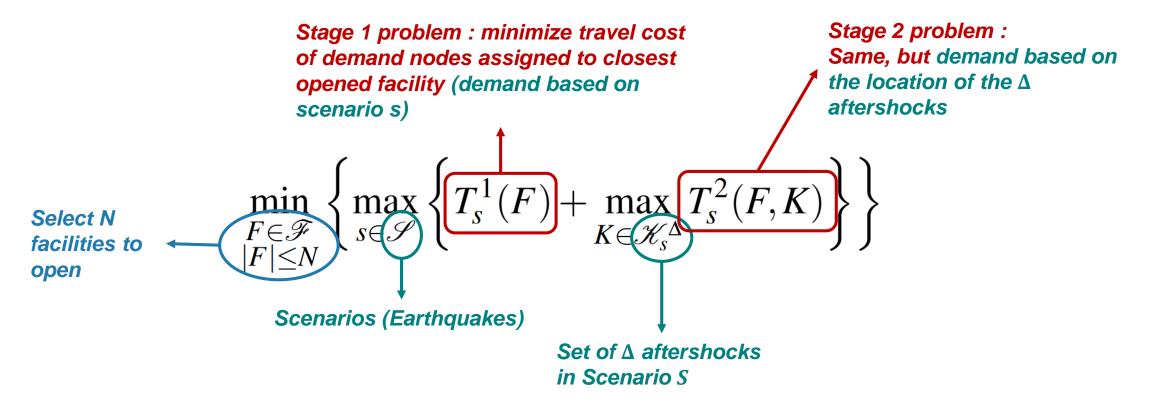
s = possible location of the main earthquake (1st stage scenario)<math>S = set of scenarios of first earthquake s K = subset of aftershocks' locations in the vicinity of s $K_s^{\Delta} = set of \Delta$ aftershocks following scenario s



Problem at a glance

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Minimize the allocation cost in the worst-case demand scenario

WE PROPOSE FOUR MODELING/SOLVING APPROACHES

1. Model Pxy-full: Full enumeration of all two-stage « aggregated » scenarios

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$$P_{xy}: \min \theta$$
subject to
$$P_{xy}: \min \theta$$

$$f(x) = \sum_{i \in \mathscr{D}} \sum_{j \in \mathscr{F}} t_{ij}(d_{\omega i}^{1} + d_{\omega i}^{2})x_{ij}$$

$$g(x) \in \Omega$$

$$f(x) = 1$$

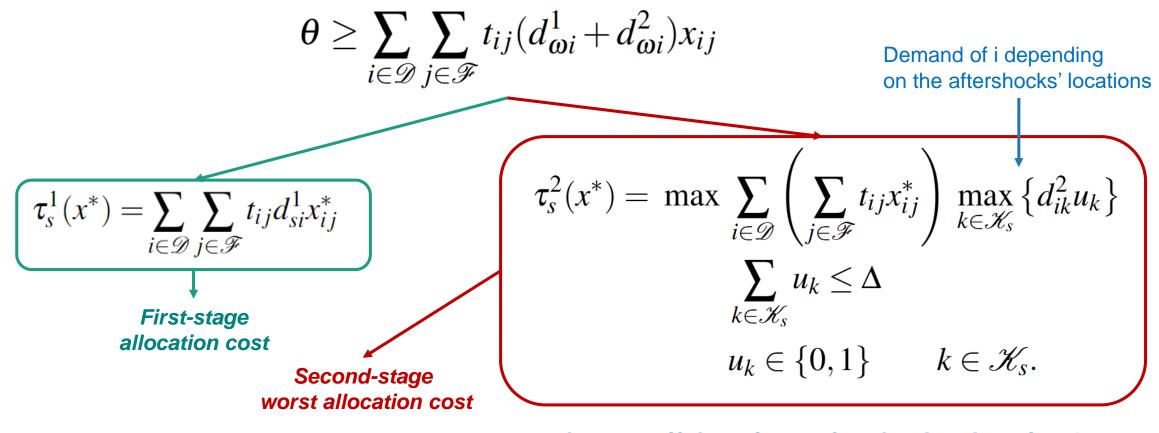
$$f(x) = 0$$

$$f(x) = 1$$

$$f(x)$$

 $x_{ij} = 1$ iff city *i* is supplied by (closest) facility *j* (opened iff $y_j = 1$)

2. Model Pxy-sep : Branch-and-Cut algorithm: Separation of aggregated scenarios



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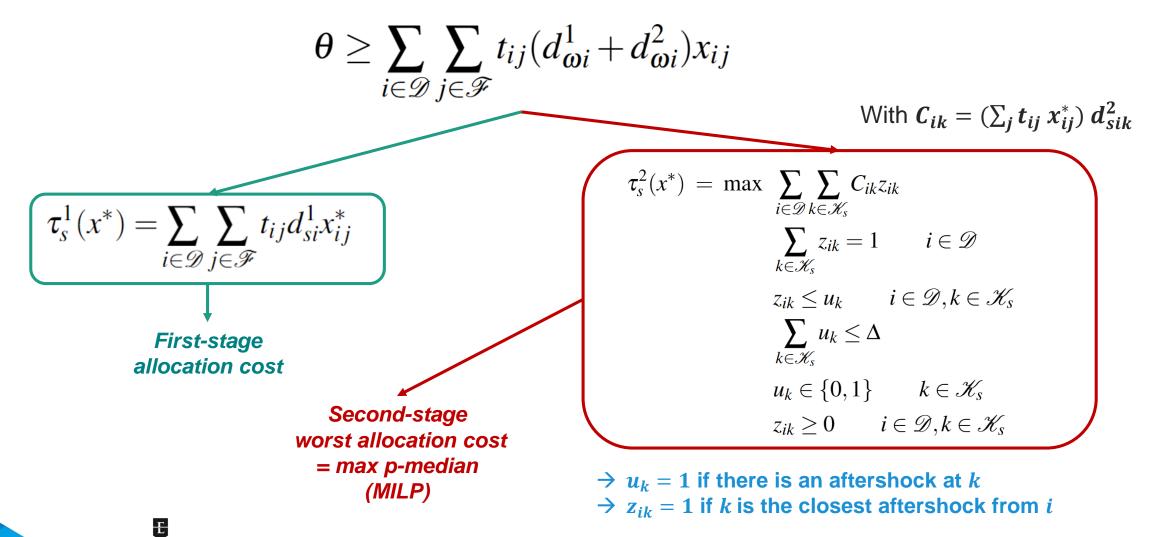
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→ $u_k = 1$ if there is an aftershock at location k→ Solvable as a MILP

2. Model Pxy-sep : Branch-and-Cut algorithm:

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Separation of aggregated scenarios : MILP reformulation of 2nd-stage



2. Model Pxy-sep :Branch-and-Cut Algorithm Cut Generation Scheme

We verify that for **each** 1st-stage (main earthquake) scenario s :

$$\boldsymbol{\theta}^* \geq \boldsymbol{\tau}_s^1(\boldsymbol{x}^*) + \boldsymbol{\tau}_s^2(\boldsymbol{x}^*)$$

If not satisfied for some *s*, we add the constraint (cut) corresponding to the violated aggregated scenario $\overline{\omega} = (s, K)$ (such *K*, or u^* vector, is easy to find, see next slide)

$$\boldsymbol{\theta} \geq \sum_{i \in \mathscr{D}} \sum_{j \in \mathscr{F}} t_{ij} (d_{i\bar{\boldsymbol{\omega}}}^1 + d_{i\bar{\boldsymbol{\omega}}}^2) x_{ij}$$



3. Model Pxy-ext : Extended Formulation (upper bound heuristic) Based on Dualization of 2nd-stage subproblem $\tau_s^2(x^*)$ LP-relaxation

$$\begin{aligned} \tau_s^2(x^*) &= \max \sum_{i \in \mathscr{D}} \sum_{k \in \mathscr{H}_s} C_{ik} z_{ik} \\ &\sum_{k \in \mathscr{H}_s} z_{ik} = 1 \quad i \in \mathscr{D} \\ z_{ik} \leq u_k \quad i \in \mathscr{D}, k \in \mathscr{H}_s \\ &\sum_{k \in \mathscr{H}_s} u_k \leq \Delta \\ &u_k \in \{0, 1\} \quad k \in \mathscr{H}_s \\ &z_{ik} \geq 0 \quad i \in \mathscr{D}, k \in \mathscr{H}_s \end{aligned}$$
With $C_{ik} = (\sum_j t_{ij} x_{ij}^*) d_{sik}^2$
 $z_{ik} = 1 \text{ iff } k \text{ is the closest} \\ aftershock from i \end{aligned}$

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Dualize with relaxed $0 \leq u_k \leq 1$

(note: we obtain an upper bound on θ^* in theory... but the LPrelaxation happened to be 0-1 on ALL instances (!!!))

$$egin{aligned} & au_s^2(x^*) = \min \; \sum_{i \in \mathscr{D}} p_{si} + \Delta \phi_s + \sum_{k \in \mathscr{K}_s} r_{sk} \ & p_{si} + q_{sik} \geq \sum_{j \in \mathscr{F}} t_{ij} d_{ik}^2 x_{ij}^* \qquad i \in \mathscr{D}, k \in \mathscr{K}_s \ & -\sum_{i \in \mathscr{D}} q_{sik} + \phi_s + r_{sk} \geq 0 \qquad k \in \mathscr{K}_s \ & q_{sik}, r_{sk}, \phi_s \geq 0, p_{si} \in \mathbb{R} \qquad i \in \mathscr{D}, k \in \mathscr{K}_s, s \in \mathscr{S} \end{aligned}$$

3. Model Pxy-ext : Extended Formulation **Final (compact) model after dualization**

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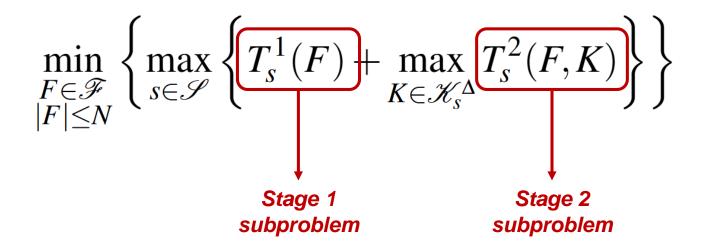
$$\begin{aligned} \mathbf{F}_{xy}^{\text{ext}} : & \min \ \theta \\ \text{subject to} \ \theta \geq \sum_{i \in \mathscr{D}} \sum_{j \in \mathscr{F}} t_{ij} d_{si}^{1} x_{ij} + \sum_{i \in \mathscr{D}} p_{si} + \Delta \phi_{s} + \sum_{k \in \mathscr{K}_{s}} r_{sk} \\ p_{si} + q_{sik} \geq \sum_{j \in \mathscr{F}} t_{ij} d_{ik}^{2} x_{ij} \quad i \in \mathscr{D}, k \in \mathscr{K}_{s} \\ -\sum_{i \in \mathscr{D}} q_{sik} + \phi_{s} + r_{sk} \geq 0 \quad k \in \mathscr{K}_{s} \\ \sum_{j \in \mathscr{F}} x_{ij} \geq 1 \quad i \in \mathscr{D} \\ \sum_{j \in \mathscr{F}} x_{ij} \geq 1 \quad i \in \mathscr{D}, j \in \mathscr{F} \\ 1 \leq \sum_{j \in \mathscr{F}} y_{j} \leq N \\ \theta \geq 0 \\ x_{ij}, y_{j} \in \{0, 1\} \quad i \in \mathscr{D}, j \in \mathscr{F} \\ q_{sik}, r_{sk}, \phi_{s} \geq 0, p_{si} \in \mathbb{R} \quad i \in \mathscr{D}, j \in \mathscr{F}, k \in \mathscr{K}_{s}, s \in \mathscr{S} \end{aligned}$$

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Adding the dualized secondstage subproblem

- **Directly solvable by a MILP solver**
- lappened to be systematically optimal good property of **p-median LP-relaxation**)

4. Formulation F_y (in the space of only y variables) Recall (general) problem :





4. Formulation F_y (in the space of only y variables): First-stage subproblem T_s^1

$$T_{s}^{1}(y^{*}) = \min_{x} \sum_{i \in \mathscr{D}} d_{si}^{1} \sum_{j \in \mathscr{F}} t_{ij} x_{ij}$$

$$\sum_{j \in \mathscr{F}} x_{ij} = 1 \quad i \in \mathscr{D}$$

$$0 \le x_{ij} \le y_{j}^{*} \quad i \in \mathscr{D}, \ j \in \mathscr{F}$$
Substitute in Primal
the assignment variables x
by a closed-loop expression in y^{*}
(possible for Uncapacitated problems)
 $p_{i} = critical \ facility \ for \ i \ (closest \ opened)$
Primal and Dual Solutions
$$x_{ij} = \begin{cases} y_{j}^{*}, & j < p_{i} \\ 1 - \sum_{j=1}^{p_{i}-1} y_{j}^{*}, & j = p_{i} , \\ 0, & j > p_{i} \end{cases}$$

$$\lambda_{i} = t_{ip_{i}} d_{si}^{1}$$

$$\mu_{ij} = \begin{cases} (t_{ip_{i}} - t_{ij}) d_{si}^{1}, & j < p_{i} \\ 0, & j \ge p_{i} \end{cases}, \quad i \in \mathscr{D}, \ j \in \mathscr{F}$$

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4. Formulation F_y : Second-stage subproblem T_s^2

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$$T_{s}^{2}(y^{*}, u_{s}^{*}) = \min_{\substack{z_{s} \\ j \in \mathscr{F}}} \sum_{\substack{j \in \mathscr{F} \\ z_{sij} \ge u_{sk}^{*} d_{ik}^{2}} i \in \mathscr{D}, k \in \mathscr{K}_{s} \qquad \text{Dualize the LP}$$

$$z_{sij} \le \widehat{D}_{i} y_{j}^{*} \quad i \in \mathscr{D}, j \in \mathscr{F}$$

$$z_{sij} \ge 0 \quad i \in \mathscr{D}, j \in \mathscr{F}.$$

$$\max_{u_{s} \in \mathscr{K}_{s}^{\Delta}} T_{s}^{2}(y^{*}, u_{s}) = \max_{\alpha_{s}, \beta_{s}, u_{s}} \sum_{i \in \mathscr{D}} \sum_{k \in \mathscr{K}_{s}} d_{ik}^{2} \alpha_{sik} - \sum_{i \in \mathscr{D}} \sum_{j \in \mathscr{F}} \widehat{D}_{i} y_{j}^{*} \beta_{sij}$$
subject to:
$$\sum_{k \in \mathscr{K}_{s}} \alpha_{sik} - \beta_{sij} \le t_{ij}, \quad i \in \mathscr{D}, j \in \mathscr{F}$$

$$\sum_{k \in \mathscr{K}_{s}} u_{sk} \le \Delta$$

$$\alpha_{sik} \le M u_{sk}, \quad i \in \mathscr{D}, k \in \mathscr{K}_{s}$$

$$\alpha_{sik}, \beta_{sij}, \ge 0, \quad u_{sk} \in \{0, 1\}, \quad i \in \mathscr{D}, j \in \mathscr{F}, k \in \mathscr{K}_{s}$$

4. Formulation F_y: Branch-and-Cut Algorithm- Cut Generation Scheme

$$\begin{split} \mathbf{F}_{\mathbf{y}} : \min \ \boldsymbol{\theta} \\ & \boldsymbol{\theta} \geq \sum_{i \in \mathscr{D}} d_{si}^{1} \left[t_{ip_{i}} - \sum_{j \in \mathscr{F}} (t_{ip_{i}} - t_{ij})^{+} y_{j} \right] + \sum_{i \in \mathscr{D}} \sum_{k \in \mathscr{K}_{s}} d_{ik}^{2} \ \boldsymbol{\alpha}_{sik}^{*} - \sum_{i \in \mathscr{D}} \sum_{j \in \mathscr{F}} \widehat{D}_{i} y_{j} \boldsymbol{\beta}_{sij}^{*}, \\ & (p_{1}, \dots, p_{|\mathscr{D}|}) \in \mathscr{F}^{|\mathscr{D}|}, (\boldsymbol{\alpha}^{*}, \boldsymbol{\beta}^{*}, u^{*}) \in \mathscr{L}_{s}^{2}, s \in \mathscr{S} \\ & 1 \leq \sum_{j \in \mathscr{F}} y_{j} \leq N, \qquad j \in \mathscr{F} \\ & \boldsymbol{\theta} \geq 0, y_{j} \in \{0, 1\} \qquad j \in \mathscr{F} \end{split}$$



4. Solution Approach P_v Branch-and-Cut Algorithm- Cut Generation Scheme

If for a given solution $(\bar{\theta}, \bar{y})$ of the master we have for some scenario \tilde{s} :

$$\bar{\theta} < \sum_{i \in \mathscr{D}} d_{i\tilde{s}}^{1} \left[t_{i\bar{p}_{i}} - \sum_{j \in \mathscr{F}} (t_{i\bar{p}_{i}} - t_{ij})^{+} \bar{y}_{j} \right] + \sum_{i \in \mathscr{D}} \sum_{k \in \mathscr{K}_{\tilde{s}}} d_{ik}^{2} \, \alpha_{i\tilde{s}k}^{*} - \sum_{i \in \mathscr{D}} \sum_{j \in \mathscr{F}} \widehat{D}_{i} \bar{y}_{j} \beta_{ij\tilde{s}}^{*}$$

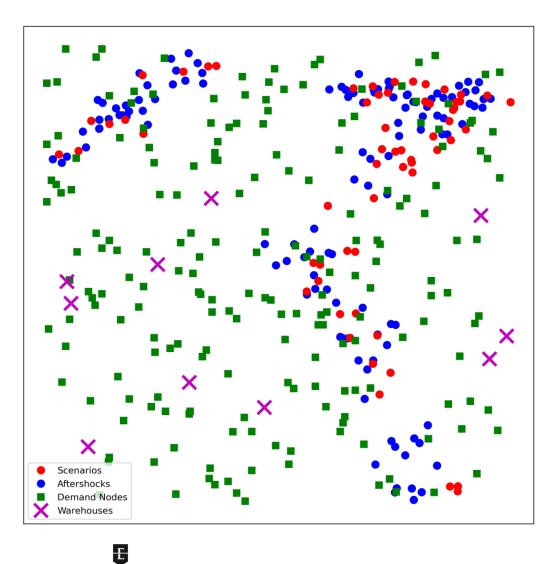
Then we add the corresponding cut for this $s = \tilde{s}$:

(note : the cut uses variables y but not x) :

$$\boldsymbol{\theta} \geq \sum_{i \in \mathscr{D}} d_{si}^1 \left[t_{ip_i} - \sum_{j \in \mathscr{F}} (t_{ip_i} - t_{ij})^+ y_j \right] + \sum_{i \in \mathscr{D}} \sum_{k \in \mathscr{K}_s} d_{ik}^2 \, \boldsymbol{\alpha}_{sik}^* - \sum_{i \in \mathscr{D}} \sum_{j \in \mathscr{F}} \widehat{D}_i y_j \boldsymbol{\beta}_{sij}^*,$$



Numerical Experiments: Generation of Instances



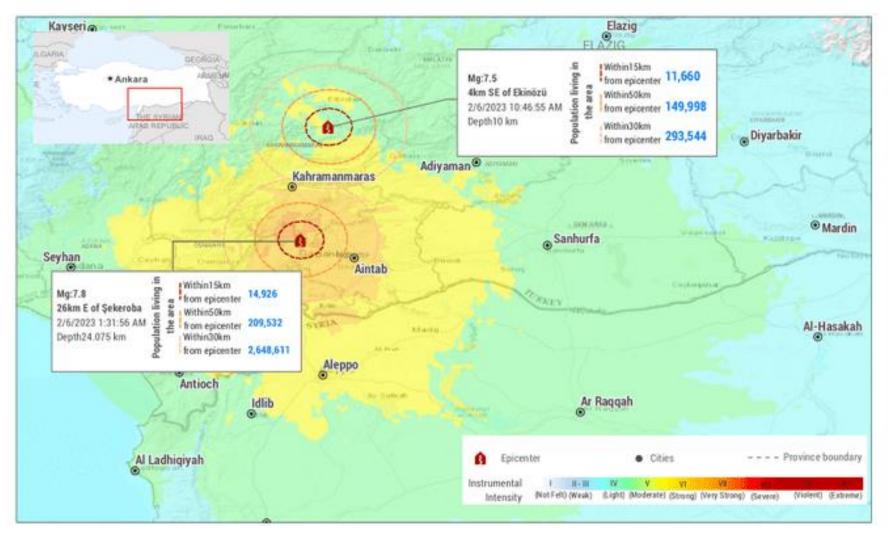
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- # demand nodes : 100 (Small), 200 (Medium), 400 (Large)
- # main earthquake scenarios s : 10, 20, 30,.., 100
- # possible aftershocks : 20, 40, ..., 200
- # possible facilities : 10 (N = 3, 5, 8)
- Δ = 3, 5, 7, 10
- Total of 360 instances

Turkey earthquake in Gaziantep on Feb 2023

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Source: OCHA, https://reliefweb.int/map/turkiye/turkiye-earthquakes-southern-turkiye-06-february-2023-06-february

Demand due to the Earthquakes and Aftershocks

Demand at any demand node is:

- Strictly increasing in the *Magnitude* of the Earthquake
- Strictly decreasing with the *Distance* from the Epicenter
- Strictly increasing to the *Population* of the Demand Node

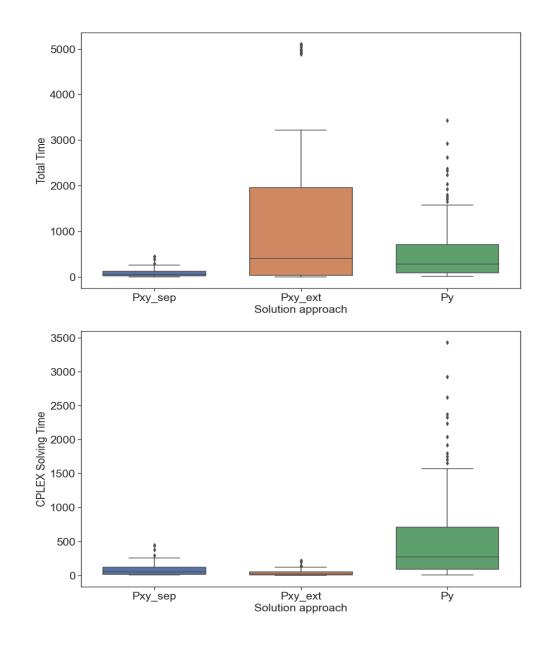
 $Demand = k \cdot \frac{Population * Magnitude^{\alpha}}{Distance^{\beta}}$

 k, α , and β are calibrated using a regression model based on data from Turkey following the earthquake of February 2023



Results

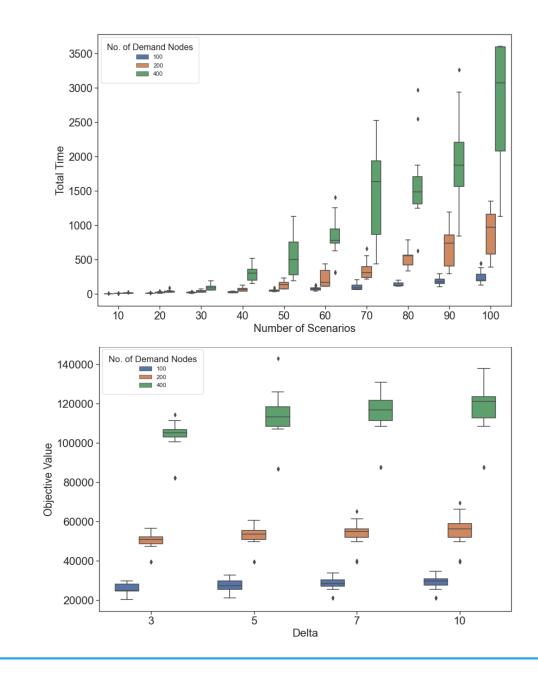
- Models were solved using CPLEX
- Initially, we solve only the Small instances to compare our models
- The « full enumeration » model could solve all instances with up to 20 main earthquakes and 40 aftershocks. Beyond that NO instance was solved for any values of N and Δ
- The solution approach P_{xy}^{sep} , with **separation of scenarios** is the **best performing** one in terms of computational time and finding an optimal solution
- P^{ext} had the worst total time. It took a lot of time for building the models by CPLEX, but once built took the lowest time
- Py performance is in-between



Results

- The separation approach was able to find a solution for all instances and optimal solutions for 356 instances out of 360 (with a low gap for others)
- The solution time increases as the number of scenarios increases

- The solution time decreases for the separation model as $\boldsymbol{\Delta}$ increases
- The objective value (allocation cost) increases with the value of Δ, particularly for instances with 400 demand nodes

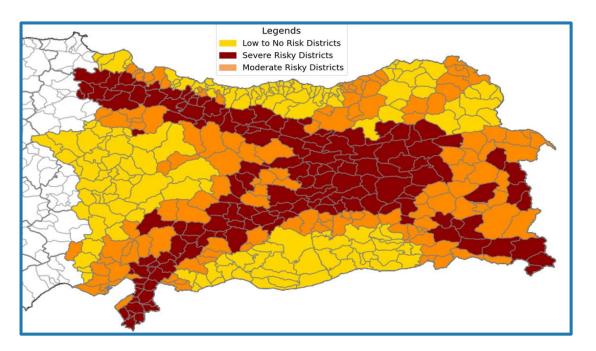


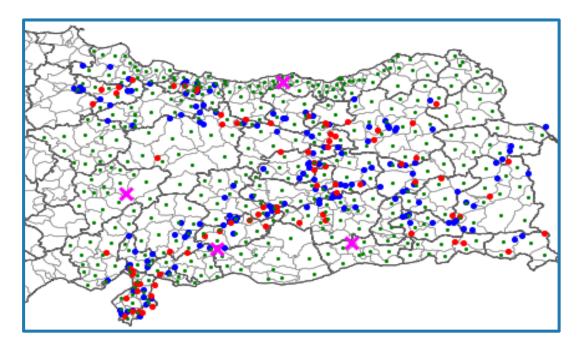
Case Study on the Turkey-Syria Earthquake of February 2023

- Implement our robust model on Turkey for establishing warehouses
- Turkey lies at the border of three tectonic plates
- 269 earthquakes between 1900 and 2023
- 20 earthquakes of magnitude greater than 7
- February 2023 earthquake is the latest one



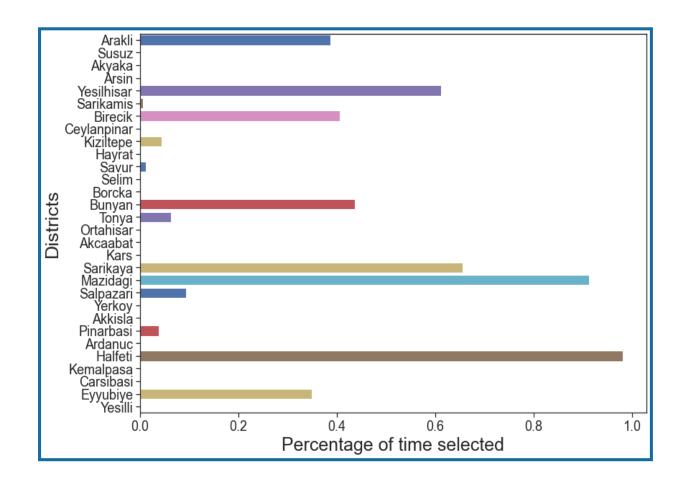
Instances for Turkey





- Instances based on the earthquake hazard map by AFAD
- Red-Severely risky, orange- moderate risky, yellow- no to low risky areas
- Red circles- earthquake epicentres, blue circles- aftershock epicenters, pink crosses- facilities, green circlesdemand nodes (420 district centers)
- 70% epicenters in the red region, 30% in the orange region

Districts selected for constructing a warehouse



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Some of the districts were selected more often than others:

- Halfeti- 98%
- Mazidagi- 91%
- Sarikaya- 66%
- Yesilhisar- 61%

Comparison with a First-Stage Model (without Aftershocks) : is it worth modelling aftershocks?

$$\begin{aligned} \mathbf{P}_{\mathbf{xy}}^{\mathbf{StageI}} : & \min \ \theta \\ & \text{subject to} \ \ \theta \geq \sum_{i \in \mathscr{D}} \sum_{j \in \mathscr{F}} t_{ij} d_{si}^1 x_{ij} \qquad s \in \mathscr{S} \\ & \sum_{j \in \mathscr{F}} x_{ij} = 1 \qquad i \in \mathscr{D} \\ & y_j \geq x_{ij} \qquad i \in \mathscr{D}, \ j \in \mathscr{F} \\ & 1 \leq \sum_{j \in \mathscr{F}} y_j \leq N \\ & \theta \geq 0, \quad x_{ij}, y_j \in \{0, 1\} \qquad i \in \mathscr{D}, \ j \in \mathscr{F} \end{aligned}$$

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$$Gap = \frac{ROV_1 - ROV_2}{ROV_2}$$

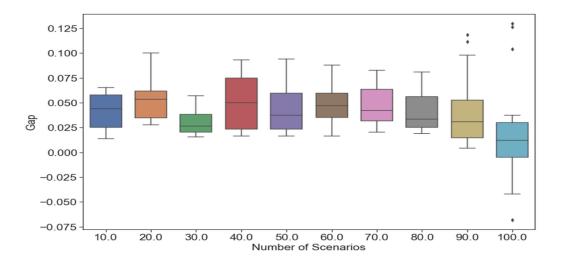
Comparison with a First-Stage Model (without Aftershocks)

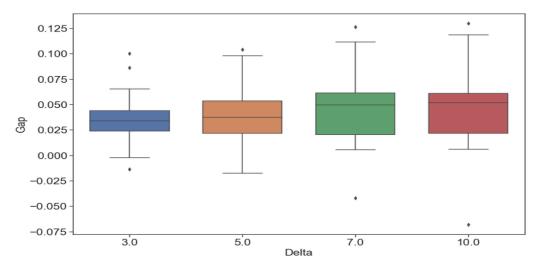
$$Gap = \frac{ROV_1 - ROV_2}{ROV_2}$$

- The gap goes over 12.5%
- Average gap is 4.5%

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Conclusions

- Robust Facility Location / SC network design Strategic planning
- <u>New</u> key feature for robust optimization: binary choices of the Δ aftershocks for the 2nd-stage uncertainty set (instead of Γ coefficients of a constraint varying in « usual » RO (≠ Bertsimas & Sim 2004)
- The model with *full enumeration of scenarios (Model 1) gets quickly intractable*
- For small-size instances, the B&C separation algorithm (Model 2) and the extended formulation (Model 3) perform similarly (up to 30 major earthquakes, 100 aftershocks, 200 demand nodes)
- As the instance size increases, the B&C separation algorithm (Model 2) is the most efficient
- The model with y variables + separation (Model 4) outperforms the extended formulation (Model 3) as the instance size increases
- The models except Model 2 are highly sensitive to the number of aftershocks
- Case Study on the Turkey Gaziantep Earthquake of 2023
- Not modelling aftershocks can give a gap on objective value up to 12.5%
- **Possible extension: capacitated version** (creates differentiation of assignments xij over the two stages)



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Thank you for your attention :)

Questions?

